Kaluza without Klein: a quantitative ab initio model for particles

Abstract

Using a Kaluza-type of model, describing the laws of electromagnetism within the formalism of differential geometry, provides a coherent, comprehensive and quantitative description of phenomena related to particles, including a convergent series of quantized particle energies with limits given by the energy values of the electron and the Higgs vacuum expectation value as well as the values for electroweak coupling constants. The geometry of the solutions for spin 1/2 defines 6 lepton-like and 6 quark-like objects and allows to calculate the fractional electric charges as well as the magnetic moments of baryons.

Electromagnetic and gravitational terms will be linked by a series expansion, the corresponding relation suggests the existence of a cosmological constant in the correct order of magnitude.

The model can be expressed *ab initio*, necessary input parameters are the electromagnetic constants.

1 Kaluza theory

Theodor Kaluza in 1919 developed a unified field theory for electromagnetism (EM) and the general theory of relativity (GR) [1] that is not suited to give properties related to particles, a problem addressed by Klein [2] who introduced the idea of compactification and attempted to join the model with the emerging principles of quantum mechanics. Therefore the theory is mainly known as Kaluza-Klein theory today. This version became a progenitor of string theory. The original Kaluza model was developed further as well [3], Wesson and coworkers elaborated a general non-compactified version to describe phenomena extending from particles to cosmological problems. The equations of 5D space-time may be separated in a 4D Einstein tensor and metric terms representing mass and the cosmological constant, Λ. Particles may be described as photon-like in 5D, traveling on time-like paths in 4D. This version is known as space-time-matter theory [4]. Both successor theories give general relationships rather than providing quantitative results for specific phenomena such as particle energy.

The model described here does not attempt to give a complete solution for a 5D theory but to demonstrate that Kaluza's ansatz provides simple, parameter-free and quantitative solutions for particles as well. Basic equations from the existing literature may be used, with one significant *simplification*:

Kaluza discovered that Maxwell's equations may be described within the formalism of GR. To get both these *and* the Einstein field equations (EFE) he needed an additional dimension and had to insert the constant of gravitation in his metric. If one settles for electromagnetic phenomena as first approximation the gravitational constant is obsolete. This does not give a unification of EM and GR, however, is a suitable ansatz to "unify" EM and particle physics. Gravitational terms can be recovered via a series expansion of the electromagnetic equations.

Curvature of space-time based on an electromagnetic version of the field equations of GR will be strong enough to localize a photon in a self-trapping kind of mechanism, yielding energy states in the range of the particle zoo. Circular polarized light is part of conventional electromagnetic theory, in the following this feature will be treated equivalently with the terms angular momentum or spin as intrinsic property of a photon and spin 1/2 will be a necessary boundary condition in the equations used.

The basic proceeding will be as follows:

Kaluza's equations for flat 5D-space may be arranged to give [4]

1) Einstein-like equations for space-time curved by electromagnetic and scalar fields,
2) Maxwell equations where the source depends on the scalar field,
3) a wave-like equation connecting the scalar Φ with the electromagnetic tensor.

To match the units of the EFE without any additional constants requires an appropriate unit system. Retaining SI units for length, time and energy the electromagnetic constants may be defined as:

$$c_0^2 = (\varepsilon_c\, \mu_c)^{-1} \qquad (1)$$

with $\varepsilon_c = (2.998\text{E+8 [m}^2\text{/Jm]})^{-1} = (2.998\text{E+8})^{-1}$ [J/m]

$\mu_c = (2.998\text{E+8 [Jm/s}^2\text{]})^{-1} = (2.998\text{E+8})^{-1}$ [s^2/Jm] .

From the Coulomb term $b_0 = e^2/(4\pi\varepsilon_0) = e_c^2/(4\pi\varepsilon_c) = 2.307\text{E-}28$ [Jm] follows for the square of the elementary charge: $e_c^2 = 9.671\text{E-}36$ [J^2]. In the following $e_c = 3.110\text{E-}18$ [J] and $e_c/(4\pi\varepsilon_c) = 7.419\text{E-}11$ [m] may be used as natural unit of energy and length. The constant G/c_0^4 [m/J] in the field equations will be replaced by $1/\varepsilon_c$:

$$(8\pi)\mathrm{G}/c_0^4 \quad => \quad \approx \ \frac{1}{\varepsilon_c} \tag{2}$$

giving:

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = \frac{1}{\varepsilon_c}T_{\alpha\beta} \tag{3}$$

Solutions of 3) for Φ in a flat 5D-metric will be used as general ansatz in a 4D-metric. This is considered to be a proof of concept, a more thorough ansatz has to be expected to incorporate angular momentum/spin into the field equations appropriately.

A solution for Φ is

$$\Phi \approx (\rho_0/r)^2 \exp(-(\rho/r)^3) \tag{4}$$

and may be seen as representing curvature of 4D space-time. Due to the derivation from a Kaluza ansatz coefficient ρ/r is a function of the electromagnetic potential, in the static approximation of this work the electric potential $\rho_0/r = e_c/(4\pi\varepsilon_c r)$. The only other parameter entering ρ will be a function of the fine-structure constant [1], α, which enters the equations through the boundary condition spin ħ/2 and its relationship with the values of the electric potential, elementary charge and electric constant.

Assuming that a 2nd term in a series expansion of EM-terms represents gravitation and should not exceed the EM-term, part of the α-terms included in ρ can be identified with the ratio of electron and Planck energy, α_{Pl}:

$$\rho_e^3 \approx (1.5)^3\sigma_0\boldsymbol{\alpha_{Pl}}\rho_0^3 \approx (1.5)^3\left[\left(\frac{\Gamma(-1/3)}{\alpha}\right)^3\right]\frac{\mathbf{1.5^2\alpha^{10}}}{\mathbf{2}}\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3 \qquad (\rho_e = \text{coefficient of electron}) \tag{5}$$

The bold term corresponds to α_{Pl}. Integrals over a function of type (4) may be given as Euler integrals, which give solutions in terms of Γ-functions, their lower integration limit, denoted σ_0 for the spherical symmetric case with a 1st approximation given in brackets, may be obtained via the boundary condition spin ħ/2 (giving σ_0 = 1.810E+8 [-]). For calculation of particle energy in col. 5 of table 1 a fit of σ_0 for and α_{Pl} according to literature values are used. The term at the right side of (5) gives a first approximation for the σ_0, α_{Pl} terms, a more detailed calculation, given in the method section, is used in col.6, table 1.

Function Φ calculated for a flat 5D-metric will be used as general ansatz in a 4D-metric:

$$g_{\mu\mu} = \left(\frac{\rho_0}{r}\right)^2\exp\left(-\left(\frac{\rho}{r}\right)^3\right),\ -\left(\frac{\rho_0}{r}\right)^2\exp\left(\left(\frac{\rho}{r}\right)^3\right),\ -r^2,\ -r^2\sin^2\vartheta \tag{6}$$

Solving for energy density and integrating gives the energy of the electron as:

$$W_e = \frac{4}{9}\frac{\Gamma(+1/3)}{(\sigma_0\alpha_{Pl})^{1/3}}\ e_c \tag{7}$$

and values for other particles as a convergent series relative to the electron state:

$$W_n/W_e \approx 3/2\frac{\alpha\wedge(1.5/3^n)}{\alpha^{1.5}}\frac{\sigma_0^{1/3}}{\sigma^{1/3}} = 3/2\,\Pi_{k=0}^{n}\alpha\wedge(\text{-}3/3^k)\ \frac{\sigma_0^{1/3}}{\sigma^{1/3}} \qquad n = \{1;2;..\} \tag{8}$$

with $\sigma_0^{1/3} \approx \Gamma(-1/3)/\alpha$ for spherical symmetry, $\sigma^{1/3} \approx (l(l+1))^{1/3}\sigma_0^{1/3}$ for the next spherical harmonics, l = 1, $\sigma^{1/3}$ = 2 Γ(-1/3)/3 for the maximum value of $\sigma^{1/3}$. Based on this, the model yields absolute particle energies with limits given by the energy values of the electron and the Higgs vacuum expectation value, see table 1.

Equating the energy terms for a point charge and a photon, hc_0/λ_C, modified by the term Φ, gives an approximation for the fine-structure constant.

$$\alpha^{-1} \approx 4\pi\,\Gamma(+1/3)\,\Gamma(-1/3) \tag{9}$$

Since this can be traced back to the product of a point charge term times a complementary integral in N-dimensions, one can obtain a single expression for both α and α_{weak}, the weak coupling constant, giving in a rest system:

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1 The relation of the masses e, μ, π with α was noted first in 1952 by Nambu [5]. MacGregor calculated particle mass and constituent quark mass as *multiples* of α and related parameters [6].

	n, l	$W_{n,Lit}$ [MeV]	α, σ-coefficients in W_n equ. (7)f	W_{calc}/W_{lit} σ_0=fit, α_{Pl}=exp	W_{calc}/W_{Lit} Method sect.	J
		≈ 1 E-7	**0**			
e^{+-}	**0, 0**	**0.51**	**$2/3\ \alpha^{-3}$**	**1.014**	**1.002**	**1/2**
μ^{+-}	**1, 0**	**105.66**	**$\alpha^{-3}\alpha^{-1}$**	**1.008**	**0.996**	**1/2**
π^{+-}	1, 1	139.57	$\alpha^{-3}\alpha^{-1}$ [$3^{1/3}$]	1.101	1.088	0
K		495				0
η^0	**2, 0**	**547.86**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}$**	**1.002**	**0.990**	**0**
ρ^0	2, 1	775.26	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3})$ [$3^{1/3}$]	1.022	1.009	1
ω^0	2, 1	782.65	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3})$ [$3^{1/3}$]	1.012	1.000	1
K*		894				1
p^{+-}	**3, 0**	**938.27**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**1.011**	**0.999**	**1/2**
n	**3, 0**	**939.57**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**1.010**	**0.998**	**1/2**
η'		958				0
Φ^0		1019				1
Λ^0	**4, 0**	**1115.68**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}$**	**1.020**	**1.008**	**1/2**
Σ^0	**5, 0**	**1192.62**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81}$**	**1.014**	**1.002**	**1/2**
Δ	**∞, 0**	**1232.00**	**$\alpha^{-9/2}$**	**1.012**	**1.000**	**3/2**
Ξ		1318				1/2
Σ^{*0}	3, 1	1383.70	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9})$ [$3^{1/3}$]	0.989	0.977	3/2
Ω^-	4, 1	1672.45	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27})$ [$3^{1/3}$]	0.982	0.970	3/2
N(1720)	5, 1	1720.00	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81})$ [$3^{1/3}$]	1.014	1.002	3/2
tau^{+-}	∞, 1	1776.82	$(\alpha^{-9/2})$ [$3^{1/3}$]	1.012	1.000	1/2
Higgs	∞,∞	1.25 E+5	$(\alpha^{-9/2})$ [$3/2\ \alpha^{-1}$] /2	1.024	1.012	0
VEV	∞,∞	2.46 E+5	$(\alpha^{-9/2})$ [$3/2\ \alpha^{-1}$]	1.042	1.030	

Table 1: Particle energies; col.2: radial, angular quantum number; col.4: α-coefficient in W_n according to (7)f; col.5/6: ratio of calculated energy, W_{calc} according to (7)f vs literature value, W_{lit} [7] – col.5 with σ_0 from fit for J_Z =1/2 and α_{Pl} given by W_e/W_{Pl} of literature values, col.6: σ_0 and α_{Pl} given by a more detailed calculation in the Methods section; col.7: angular momentum J_z [ħ]; Only states given in [7] as 4-star, characterized as *„Existence certain, properties at least fairly well explored"*, are included, up to Σ'^0 all states given in [7] are listed; blanks in the table are discussed in chpt. 10 of the Methods section.

$$\alpha_N^{-1} = S_N \frac{\Gamma(+(N-2)/N)\,\Gamma(-(N-2)/N)}{(N-2)^2} \qquad N = \{3;4\} \tag{10}$$

Equation (10) allows to give ρ and consequently particle energy in terms of elementary charge, electric constant and mathematical constants only (table 2).

Dimension – space	coupling constant	Value of *inverse* of coupling constant, α_N^{-1}	
4D	$\alpha_4 = \alpha_{weak}$	$2\pi^2\,\Gamma_{+1/2}\,\Gamma_{-1/2}/4 = \pi^3 =$	31.0
3D	$\alpha_3 = \alpha$	$4\pi\,\Gamma_{+1/3}\,\Gamma_{-1/3} = 4\pi\,\Gamma_{+1/3}\,\Gamma_{-1/3} =$	136.8

Table 2: Values of electroweak coupling constants

2 Quaternion ansatz

The Kaluza model with spin 1/2 as boundary condition described above allows to calculate ab initio values for the free parameters of the standard model of particle physics (SM). Reversing the main focus, emphasizing angular momentum and implicitly assuming curvature of space as necessary boundary condition for localization allows to reproduce the fermion particle content of the SM and to explain phenomena that are related to properties of quarks.

A circular polarized photon with its intrinsic angular momentum interpreted as having its E- and B-vectors rotating around a central axis of propagation will be transformed into an object that has the - still rotating - E-vector constantly oriented to a fixed point [2], the origin of a local coordinate system. The vectors E, B and C of the propagation velocity are supposed to be locally orthogonal and subject to standard Maxwell equations. This has the following qualitative consequences:

1) Such a rotation is related to the group *SO(3)* (and *SU(2)* as important special case). A quaternion ansatz will be used for modeling the respective rotations.
2) E-vector constantly oriented to a fixed point implies *charge*. As implicitly assumed above, neutral particles are supposed to exhibit nodes separating corresponding equal volume elements of reversed E-vector orientation and opposite polarity.
3) A local coordinate system = rest system implies *mass*.
4) In case of any lateral extension of the E-field, for r -> 0 the overlap of a rotating E-vector implies rising energy density, resulting in *rising curvature of space-time* according to GR or its modification as of equ. (3).
5) The orthogonal vectors E, B and C can be given in 2 different *chiral* states (left- right-handed).
6) As essentially electromagnetic waves such states are consistent with a “point-like” structure function on the other hand imply a spatial distribution of energy density and angular momentum / spin.

For quantitative results 3 orthonormal vectors E, B, C, each described as imaginary part of a quaternion with real part 0, will be subject to alternate, incremental rotations around the axes E, B and C. In the following only solutions where one of the incremental angles of rotation has half the value of the other two will be considered. This may serve as a primitive model for spin J=1/2. There are 3 possible solutions corresponding to half the angular frequency for each of the components E, B, or C, respectively. The trajectory of the E-vector encloses a spherical cone, the spherical cap of the cone encompasses a fraction of the area of a hemisphere of 2/3, 1/3 and 1/3, respectively. Mirroring at the center of rotation gives the equivalent double cone (blue in fig.1), the fractions of both caps in relation to the surface of the total sphere may be interpreted to give partial charges of 2/3, 1/3 and 1/3 according to Gauss’ law. It is suggestive to identify such components with uds-quarks. In the following the assignment (half-frequency-E-rotation, charge +2/3, U), (half-B, charge -1/3, D), (half-C, charge -1/3, S) will be used. The magnetic moments of the uds-baryons will be calculated from appropriate combinations of these UDS-objects in 3.2.

The E-vector might as well be interpreted to enclose the complement of the double cone of a 3D-ball (white in fig.1), to be called a spherical wedge in the following. This gives the objects complement-U, complement-D, complement-S with charges 1/3, 2/3, 2/3. These objects may be attributed to c, b, t-quarks [3].

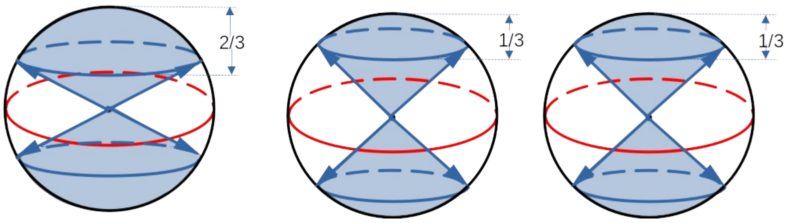

Fig.1: Trajectories of the E-vector, enclosing spherical cones and toroidal wedges

Such entities of (single) spherical cones and toroidal wedges may be used as elementary building blocks to be combined to form more complex objects of still orthogonal E(t) and B(t) fields, pending on fitting phase / angular momentum and chirality as well as interference of the fields itself. A mismatch in phase/ chirality may result in nodal planes and higher energy states. In the following it will be left undecided if more complex compositions of such objects might be interpreted to represent time averages of propagating EBC-vectors or a standing wave.
A combination of two cones to give a spherical double cone will always give a valid solution with any spin or chirality and is considered to give approximate y_1^0 solutions.

The simplest objects will result from combining an U, D or S-component with its complement of same phase and chirality, which formally will reassemble the complete sphere. These objects may be attributed to the charged and neutral leptons. An electron might be considered e.g. as an (anti-U + (U-Complement = B))

2 This holds for B (and C) as well, i.e. for r < λ_C both E- and B-fields will be charge-like.
3 It may be expected that their more extended geometry might be less favourable in a combination for hadrons, leading to higher energy states.

particle, however, unlike a B-meson with spin 1/2. While this is not possible with quarks, i.e. objects with particle character, this gives the simplest state for an electromagnetic wave with the lowest energy.
Composite objects - in particular if composed of 3 UDS-components – may feature sufficient spherical symmetry to conform to the respective energy equation (8).

3.2 Magnetic moments of baryons

A crucial test for the applicability of such an ansatz is the calculation of magnetic moments. A combination of 3 orthonormal UDS-components whose half-integer spin-components add up to yield the appropriate total spin J_z = 1/2 will be used. The calculated moments are a product of a component depending on total energy/mass expressed by a term for a current loop containing the Compton wavelength, and an average of the B-field calculated with quaternions (B_avg). Table 3 gives the ratio of calculated and experimental values [7] [4]. Since λ_C may be calculated according to the methods given above these values are *ab initio* as well, however, for greater accuracy values of λ_C according to [7] are used.

		λ_C	e c_0 *λ_C /2	B_Avg	|M|Calc = $ec_0\lambda_C B_{avg}/2$	|M|Exp[Am^2]	|M|Calc/ |M|Exp	|M|Calc/|M|Exp Const. quark
p^{+-}	UUD	1.32E-15	3.17E-26	0.440	1.39E-26	1.41E-26	0.988	-
n	DDU	1.32E-15	3.17E-26	0.301	9.55E-27	9.66E-27	0.988	0.973*
Λ^0	UDS	1.10E-15	2.64E-26	0.111	2.94E-27	3.10E-27	0.949	-
Σ^+	UUS	1.04E-15	2.50E-26	0.497	1.24E-26	1.24E-26	1.002	1.090
Σ^-	DDS	1.04E-15	2.50E-26	0.234	5.83E-27	5.86E-27	0.994	0.897
Ξ^0	USS	9.43E-16	2.26E-26	0.267	6.05E-27	6.31E-27	0.958	1.152
Ξ^-	DSS	9.38E-16	2.25E-26	0.134	3.01E-27	3.06E-27	0.983	0.784

Table 3: Magnetic moments for UDS-Baryons [4]; col.3: Compton wavelength [7]; col.4: magnetic moment for current loop; col.5: average B-component from quaternion calc.; col.6: calculated magnetic moments; col.7: values from experiment [7]; col.8: ratio calculated / experiment value; col.9: ratio (calculated constituent quark model, [7]) / experiment [7]) added for reference.

Minor systematic errors have to be expected in this model. The ratio of magnetic moments for pairs of particles from the same family gives:

	|M|Calc (Col.6)	B_avg (Col.5)	Const. Quarks
M(p/n)_Calc/M(p/n)_Exp	0.999809	1.001187	0.973*
M(Σ^+/Σ^-)_Calc/M(Σ^+/Σ^-)_Exp	1.007813	1.001111	1.115
M(Ξ^0/Ξ^-)_Calc/M(Ξ^0/Ξ^-)_Exp	0.974652	0.969601	1.470

Table 4: Ratio of particle magnetic moments of baryon pairs compared for calculated and experimental values [7] (col.3: geometry only, B_avg; col.2 inc. exp. particle energy); col.4: values based on constituent quarks [7].

The solutions for nucleons, are distinguished by the exchangeability of U- and D-components, as well as one U and one D-component occupying approximately the same space [5], indicating a particular stable configuration involving oppositely charged components.

The orthonormal EBC-vectors feature two possible chiral configurations, right-handed “R” and left-handed “L”. This suggests to be a possible source for a factor 3 frequently appearing in the quantitative interpretation of processes involving a quark-antiquark-pair. While this is attributed to the 3 “colors” of quarks in the SM, the same factor would result for any UDS-pair having the possibility to exist in triplet-like states, “LL”, “RR” and 1/√2 (LR+RL) [6] (referring to an axial vector representing the EBC-configuration).

4 Gravitation and cosmological constant

In this work the expression

$$b_0 = G\, m_{Pl}^2 = G\, W_{Pl}^2 / c_0^4 \qquad (11)$$

4 To allow for comparison with tabulated values of M in units of [Am^2] the calculations in this chapter use e [C] not e_c [J], conversion factor: [m^2C/s] /[$m^2 J/s$] = e/e_c = 1/19.4 [C/J]. Only absolute values will be considered.
5 Time average! All E,B-components involved are orthogonal at any given point in time. Additional aspects concerning nucleons see method section chpt. 11.
6 With a singlet state corresponding to destructive interference; Particles and antiparticles are supposed to have opposite chirality, however, within this model there exists no fundamental reason that prevents any combination for charge / spin / chirality.

is used as definition for Planck terms. The constant G may be given as:

$$G \approx \frac{\alpha_{Pl}^2 c_0^4 b_0}{W_e^2} \tag{12}$$

Since W_e may be expressed as function of π, $\Gamma_{+1/3}$, $\Gamma_{-1/3}$ and e_c only, G may be expressed as a coefficient based on electromagnetic constants, $G \approx 2/3\ c_0^4 \alpha^{24}/(4\pi\varepsilon_c)$.

Terms for gravitation may be recovered via a series expansion of the exponential in (4), giving for the first two terms of energy density:

$$w \approx \frac{\varepsilon_c \rho_0^2}{r^4} e^{\nu} \approx \varepsilon_c E^2 \left[1 - \sigma\alpha_{Pl} \left(\frac{e_c}{4\pi\varepsilon_c \mathbf{r}} \right)^3 \right] \tag{13}$$

which is a very good approximation for $r > \alpha\lambda_C$. The 1st term is the classical Coulomb term for energy density. The 2nd term contains by definition the ratio between Coulomb and gravitational terms for *one* particle, α_{Pl}. To turn this into the exact Coulomb / gravitation relationship requires that **r in the exponential** may not be considered to be a free parameter for $r > \lambda_C$, the limit of a real solution [7], and it has to be assumed that spin does not play a role for $r >> \lambda_C$. This would give the general expression for a series expansion as:

Coulomb-term $(1 - \alpha_{Pl})$.

Equivalent terms for potentials or the vacuum (methods, chpt.3) may be constructed as well.

A metric of type (6) offers the possibility to produce additional minor terms not relevant for particle energy. G_{00} will in general contain terms such as $\rho_n^3/\mathbf{r}^5$ or $\rho_n^6/\mathbf{r}^8$ with all r originating from derivatives of the exponential only. Using $\mathbf{r} = e_c/(4\pi\varepsilon_c)$ as upper bound of r, as suggested above will yield approximate values in the order of magnitude of critical, vacuum density, ρ_c, ρ_{vac} and of Λ:

$$\frac{\Phi''}{\Phi} \approx \frac{\rho^3}{\mathbf{r}^5} \approx \frac{\alpha_{Pl}}{(e_c/(4\pi\varepsilon_c))^5} \left(\frac{e_c}{4\pi\varepsilon_c} \right)^3 = \alpha_{Pl} \left(\frac{4\pi\varepsilon_c}{e_c} \right)^2 = 0.089\ [\mathrm{m}^{-2}] \tag{14}$$

Multiplied by ε_c this gives an energy density of 2.97E-10 [J/m^3].

Multiplied by the conversion factor for the electromagnetic and gravitational equations, equ. (2), (14) gives as estimate for Λ:

$$\alpha_{Pl} \frac{(4\pi)^2 \varepsilon_c^3}{e_c^2} \frac{8\pi G}{c_0^4} \approx \quad 6.17\text{E-53}\ [\mathrm{m}^{-2}] \tag{15}$$

5 Relationship with concepts of quantum mechanics

It is a common thought that the theory of general relativity (GR) has to be unified with quantum mechanics (QM). The model presented here suggests that the ansatz of Kaluza, in particular in combination with the boundary condition spin 1/2, is sufficient to give an excellent model for particles, bypassing QM in 1st approximation. The major deviation from conventional GR is dropping the constant of gravitation in the field equations, a minor thing from a mathematical point of view. The resulting objects of interest are waves only, which naturally fits basic concepts of QM. Other general features of quantum mechanics that emerge from such an ansatz include quantization of energy or the pivotal constant of quantum mechanics, Planck's constant, h, that may be derived from the electromagnetic constants and geometry as expressed in the derivation of α.

While the integrals of the Kaluza ansatz allow to calculate the free parameters of the SM and to remove some values from the set of fundamental constants:

electromagnetic constants, h, G, α, α_{weak}, energies of elementary particles => electromagnetic constants,

the quaternion ansatz reproduces the content of “elementary” fermions of the SM. The model can explain the number of “elementary” objects, based essentially on the possibility to have 3 orthogonal vectors in 3D-space, and allows to calculate their (partial) charges and magnetic moments. Leptons are an integral part of the particle classification scheme.

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7 The existence of an integration limit implies that the exponential in (4) has to be an approximation of e.g. a damped oscillation-like solution.

There are several features of the model indicating a close relationship with electroweak theory: chirality, SO(3), SU(2) symmetry of the individual particles as well as a rotational symmetry between particles [8], the energy of the Higgs boson /VEV as upper limit for particle energy [9] and last not least the possibility to calculate the IR-limits of the electroweak coupling constants. On the other hand, there seems to be no deeper connection with the concepts of QCD, such as color or gluons. Properties such as confinement or the need for adhering to the Pauli principle in e.g. the Δ^{++} are obsolete from the outset for an object that is basically a (5D-) electromagnetic wave. The development of the SM from constituent quarks towards QCD, based on valence and sea quarks plus gluons, was in part required by the limitations in explaining some scattering experiments with 3 point-like objects only. The waves of this model are consistent with a point like structure function and still feature spatial extension from the outset.

QED terms are considered to be a necessary amendment for this model. The deviation of calculated results from the experimental values is typically in the range 0.01 - 0.001, consistent with the non-linearity of the Γ-functions and a variation of input parameters related to elementary charge in an order of magnitude of QED corrections.

References

[1] Kaluza, T., "Zum Unitätsproblem in der Physik". Sitzungsber. Preuss. Akad. Wiss. Berlin. 966–972 (1921)
[2] Klein, O., "Quantentheorie und fünfdimensionale Relativitätstheorie", Zeitschrift für Physik A. 37 (12), 895–906 (1926); doi:10.1007/BF01397481
[3] Wesson, P.S., Overduin, J.M., arxiv.org/abs/gr-qc/9805018v1 (1998)
[4] Wesson, P.S., Overduin, J.M., "Principles of Space-Time-Matter", Singapore, World Scientific (2018)
[5] Nambu,Y. Progress of theoretical physics 7, 595-596; 1952
[6] MacGregor, M., "The power of alpha", Singapore, World Scientific (2007)
[7] Workman, R.L. et al., Particle Data Group, “REVIEW OF PARTICLE PHYSICS”, Prog. Theor. Exp. Phys. 2022, 083C01 (2022); https://doi.org/10.1093/ptep/ptac097; https://pdg.lbl.gov/

8 Either considering the transformation of the half-rotation mode between E,B,C-components or the transformation between E- and B-fields e.g. in the pair UD.
9 Some speculative relationship with electroweak bosons is given in chpt. 10 of the method section.