Theodor Kaluza's Theory of Everything: revisited

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Abstract
Using a Kaluza-type of model, describing the laws of electromagnetism within the formalism of curved 4D-/flat 5D-space-time, provides a coherent, comprehensive and quantitative description of phenomena related to particles, including a convergent series of quantized particle energies, with limits given by the energy values of the electron and the Higgs vacuum expectation value, and the values for electroweak coupling constants. The geometry of the solutions for spin 1/2 defines 6 lepton-like and 6 quark-like objects and allows to calculate their fractional electric charges as well as magnetic moments of baryons.
A series expansion yields electromagnetic terms, gravitational terms and a de Sitter background that provides a link between the cosmological constant and the a_0 parameter of MOND/BTFR, giving both values in the correct order of magnitude.
The model can be expressed *ab initio*, necessary input parameters are the electromagnetic constants.

1 Introduction

Theodor Kaluza in 1919 developed a unified theory of gravitation and electromagnetism that produced the formalism for the field equations of the general theory of relativity (GR) and Maxwell's equations of electromagnetism (EM) thus unifying the major forces known at his time. His 5-dimensional model [1] is not suited to give properties related to particles, a problem addressed by Oskar Klein [2] who introduced the idea of compactified extra dimensions and attempted to join the model with the emerging principles of quantum mechanics. Therefore the theory is mainly known as Kaluza-Klein theory today and in this version became a progenitor of string theory. The work presented here does not follow this path but rather the extension of Kaluza's original work put forth among others by Wesson and coworkers [3], [4] in a version known as space-time-matter theory. In this concept one makes use of Campbells theorem which states that any curved N-D space can be embedded at least locally in a flat (N+1)-D-(i.e. Minkowski-) space. Using either 4D-curved or 5D-flat space-time is merely a mathematical choice, there is nothing "extra" in terms of physics. This is analog to the approach for a 4D de Sitter space (dS_4) which represents a submanifold of the 5D-space-time discussed in the Kaluza context and provides a useful link between particle and cosmological phenomena.

A crucial *simplification* of Kaluza's original ansatz concerns the constant of gravitation, which he introduced ad hoc in his metric as a coefficient to render the EM potentials dimensionless and get the field equations of GR with the appropriate constant. This is a rather unfitting combination and in this work G will not be part of the metric. Gravitation can be recovered quantitatively by a series expansion and G can be expressed as an EM-term.

Curvature of space-time based on an electromagnetic version of the field equations of GR will be strong enough to localize a photon in a self-trapping kind of mechanism and in combination with a boundary condition, spin of a fermion, will yield accurate energy states in the range of the particle zoo. Circular polarized light is part of conventional electromagnetic theory, in the following such a feature will be treated equivalently with the terms angular momentum or spin [1] as intrinsic property of particles. Spin will be a necessary boundary condition to determine an integration limit for the equations used. Since at this point there is no obvious ansatz for integrating spin into the metric of this model, any metric discussed in the following should be considered as an approximation only [2].

The basic proceeding will be as follows:

Following [4] Kaluza's equations will be elaborated for flat 5D-space-time. They may be arranged to give (cf. [4], chapter 6.6):

1) Einstein-like equations for space-time curved by electromagnetic and scalar fields (equ. (3)),
2) Maxwell equations where the source depends on the scalar field,
3) a wave-like equation connecting the scalar Φ with the electromagnetic tensor (equ. (4)).

1 "Spin" will be used as a generic term not necessarily implying specific features of the quantum mechanical term.
2 dS_4 may cover some aspects of spin, but does not give a direct relationship to S_z=1/2.

Solutions of 3) for Φ in a flat 5D-metric will be used in a general ansatz for a metric. Due to 3) Φ includes a term with an exponential function of the EM-potentials, in the approximation of this work the electric potential, A_{el}. The only other coefficient entering the equations will be ħ of the boundary condition spin. These coefficients are related by the fine-structure constant, α, which consequently will play an important role in this work, and Φ can in general be given as function of A_{el} and α, see chpt. 2.4. The α-terms in the exponential part of Φ will be given as two coefficients:

- Coefficient **σ** represents angular momentum of particle states and is the decisive parameter for providing an integration limit/length parameter [3].
- Coefficient **ß** [4], serves as a ground state coefficient for particles and together with σ determines the particle energy spectrum. It will be the only remaining parameter beyond the particle radius and as such plays an important role for particle interaction.

Both σ and ß can not only be referred to well known experimental values but also approximately derived from fundamental considerations (see e.g. [A4]) or they can be converted into each other (chpt 2.4.4.1). Therefore, all results of this model are considered to be calculable *ab initio*, using electromagnetic and mathematical constants only.

The exponential function of Φ is crucial in obtaining the following results:

- It allows to integrate over r^{-2} of a point charge and yields an energy in the range expected for neutrinos when inserting σ only and a set of converging series of particle energies as function of α [5] with limits given by the energy of the electron and the Higgs vacuum expectation value (VEV) when inserting both σ and ß.
- Series expansion of the exponential will leave only the ground state coefficient ß as particle related parameter in higher order terms.
  - Within the accuracy of the calculations coefficient ß is identical with the ratio of the electron and Planck energy, (cf. chpt. 3.1) and the 2nd order term can be identified with gravitation, as expectable.
  - The 3rd order term can be identified with deviations from basic gravitation and yields values for the cosmological constant, Λ_0, the Baryonic-Tully-Fisher-Relation (BTFR) of galaxy rotation curves and the related coefficient a_0, discussed extensively in the context of MOND-models, in the correct order of magnitude. This may be interpreted in terms of a de Sitter space (cf. chpt. 3.3).

Focusing on the angular momentum aspects of the model, in chpt. 4 the rotation of a set of orthogonal E, B, C-vectors, attributed to the electromagnetic fields and the propagation with the speed of light, C, will be modelled via quaternions. This gives 3 possible solutions for spin 1/2 defining 6 distinct geometric objects with partial charges of 1/3 and 2/3. Combining 2 complementary solutions gives 6 lepton-like entities as the simplest, node-free case, combining 3 appropriate solutions allows to calculate magnetic moments of baryons.

Typical accuracy of calculations is in the order of 0.0001. However, ambiguities involving the incomplete gamma-functions may be in the range of a few percent (see e.g. [A3.4]). The deviation of calculated results from the experimental particle values is typically in the range of 0.01 - 0.001 and might still be consistent with a variation of input parameters related to elementary charge in an order of magnitude of QED corrections (cf. chpt. 2.5.4).
To focus on the more fundamental relationships some minor aspects of the model are exiled to an appendix, related topics will be marked as [A].

1.1 System of natural units
In the following a unit system based on SI for units of mechanics but with modified EM-units, indicated by subscript c, will be used that is particularly suited for simplifying the relevant expressions:

$$c_0^2 = (\varepsilon_c \mu_c)^{-1} \qquad (1)$$

with $\varepsilon_c = (2.998E+8\ [m^2/Jm])^{-1} = (2.998E+8)^{-1}\ [J/m]$
$\mu_c = (2.998E+8\ [Jm/s^2])^{-1} = (2.998E+8)^{-1}\ [s^2/Jm]$.

From the Coulomb term $b_0 = e^2/(4\pi\varepsilon_0) = e_c^2/(4\pi\varepsilon_c) = 2.307E-28$ [Jm] follows for the square of the elementary charge: $e_c^2 = 9.671E-36\ [J^2]$. In the following $e_c = 3.110E-18$ [J] and $r_l = e_c/(4\pi\varepsilon_c) = 7.419E-11$ [m] may be used as natural unit of energy and length [6].

3 It is assumed that in general particle states feature $S_Z = 1/2$ or are composed of spin 1/2 components.
4 Denoted α_{Pl} in earlier versions of this work.
5 The relation of the masses e, μ, π with α was noted first in 1952 by Nambu [5]. MacGregor calculated particle mass and constituent quark mass as *multiples* of α and related parameters [6].
6 This is the only unique choice for a unit system which gives dimensionless units for A_{el} in the metric and would be the default choice in a system of natural units. In SI proper a constant k=0.00515[C/J] would be required for ke/(4πε) [m].

2 Calculation of energies

2.1 Kaluza theory

Kaluza theory is an extension of general relativity to 5D-space-time with a metric given as [cf. 4, equ. 2.2]:

$$g_{AB} = \begin{bmatrix} (g_{\alpha\beta} - \kappa^2 \Phi^2 A_\alpha A_\beta) & -\kappa \Phi^2 A_\alpha \\ -\kappa \Phi^2 A_\beta & -\Phi^2 \end{bmatrix} \quad (2)$$

In (2) roman letters correspond to 5D, Greek letters to 4D. κ corresponds to a general constant appropriate for an EM unit system that turns κA_α into a dimensionless quantity. To get the field equations of GR Kaluza assigns k to a gravitational term. Assuming 5D space-time to be flat, i.e. $G_{AB} = 0$, gives for the 4D-part of the field equations [cf. 4, equ. 2.3]:

$$G_{\alpha\beta} = \frac{\kappa^2 \Phi^2}{2} T_{\alpha\beta}^{EM} - \frac{1}{\Phi} \left(\nabla_\alpha (\partial_\alpha \Phi) - g_{\alpha\beta} \Box \Phi \right) \quad (3)$$

From $R_{44} = 0$ follows:

$$\Box \Phi = -\frac{\kappa^2 \Phi^3}{4} F_{\alpha\beta} F^{\alpha\beta} \quad (4)$$

2.2 Modification of Kaluza's metric

In this work the focus will be on EM-terms only, i.e. $g_{\alpha\beta}$ of (2) is set to zero. Moreover, only an electrostatic approximation, i.e. the electric potential, $A_{el} = e_c/(4\pi\varepsilon_c r) = \rho_0/r$ [-] will be considered. In the following ρ_0 will refer to the A_{el}-term, while dropping subscript 0 will indicate a general solution where ρ may contain additional terms.

This is an approximation not only in neglecting contributions of the magnetic potentials but also in not considering spin. It is therefore not possible to give an *exact* metric for the problems considered here. [A2] introduces 2 versions for illustrative purposes. The following approximations will be used.

Only derivatives with respect to r of a spherical symmetric coordinate system will be considered.

According to Campbell's theorem [4] a flat (N+1)-D metric is mathematically equivalent to a curved N-D metric [7] so both approaches are compatible in principle. Solutions of (4) for an approximate Φ of a flat 5D-metric will be used as general ansatz in a 4D-metric. In

$$\Phi_N \approx \left(\frac{\rho}{r} \right)^{N-1} \exp\left(-\left(\frac{\rho}{r} \right)^N / 2 \right) \quad (5)$$

the term of highest order of exponential N, given by $\Phi'' \sim \rho^{3N-1} / r^{3N+1}$, may be interpreted to provide terms for

$$A_{el}(r) = e_c/(4\pi\varepsilon_c r) = \rho_0/r \sim \rho/r \quad (6)$$

see [A1]. The significance of (5) lies in providing the relationship of exponential and pre-exponential terms and first of all in the requirement to contain powers of (ρ_0/r) in the exponent of Φ_N.

The difference in order of magnitude between ρ_0 and ρ, to be elaborated on below, results in the leading term for particle energy being due to the Christoffel symbols of the angular coordinates [8], yielding a solution for particle energy that is essentially independent from minor details of the metric, including the use of either a 4D- or a 5D-metric.

Concerning dimensions and unit systems:

Since E has to be the derivative of a unitless κA_{el}, $(\kappa E)^2$ will have appropriate units for T_{00} (with chpt. 1.1: $E^2[1/m^2]$). An expression with energy density, w, in T_{00} would require an expansion with some appropriate coefficient for an electric constant, ε, turning the square of the electric field into energy density, $(\varepsilon\kappa^2 E^2)$, which in turn requires ε to cancel in T_{00}: $T_{00} = 1/\varepsilon\ (\varepsilon\kappa^2 E^2) = 1/\varepsilon\ w$.

When equating G_{00} with T_{00} the G-term in the conventional field equation will be replaced by 1/ε, giving

$$(8\pi) G / c_0^4 \quad => \quad \approx \quad 1/\varepsilon \quad (7)$$

in

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = -\frac{1}{\varepsilon} T_{\alpha\beta} \quad (8)$$

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7 At least locally, however, classic GR is a local theory as well.

8 Giving a "-1" component in the Ricci-tensor; This is consistent with curvature being due to the lateral extension of the E-vector in the quaternion ansatz of chpt. 4;

The relation between the gravitational and the EM scale will be given by a numerical factor of

$$8\pi\varepsilon_c G/c_0^4 = 6.927\text{E-52} = 2\gamma_\#^2 = 2\cdot(1.861E-26)^2 \tag{9}$$

2.3 Point charge energy

A broad class of metrics will give (10) as solution for G_{00} (cf.[A2]):

$$G_{00} = -\rho_0^2/r^4 \exp(-(\rho/r)^3) = w/\varepsilon_c \tag{10}$$

The exponential of function Φ allows to integrate the electric field of the point charge. The volume integral over the energy density of (10) gives the energy of particle n according to:

$$W_n = \varepsilon_c \rho_0^2 \int_0^{r_n} \exp(-(\rho_n/r)^3) r^{-4}\ d^3r = 4\pi\varepsilon_c\rho_0^2 \int_0^{r_n} \exp(-(\rho_n/r)^3) r^{-2}\ dr \tag{11}$$

Solutions for integrals over $\exp(-(\rho/r)^3)$ times some function of r can be given by:

$$\int_0^{r_n} \exp(-(\rho_n/r)^N) r^{-(m+1)} dr = \Gamma(m/N, (\rho_n/r_n)^N)\ \frac{\rho_n^{-m}}{N} = \int_{(\rho_n/r_n)^N}^{\infty} t^{\frac{m}{N}-1} e^{-t} dt\ \frac{\rho_n^{-m}}{N} \tag{12}$$

in this work used for N = {3; 4}, m = {-2; -1; 0; +1;+2}. The term $\Gamma(m/N, (\rho_n/r_n)^N)$ denotes the upper incomplete gamma function, given by the Euler integral of the second kind [9]. In the range of values relevant in this work, for m/N ≥ 1 the complete gamma function $\Gamma_{m/N}$ can be a sufficient approximation, for m/N ≤ 0 the integrals have to be calculated numerically, requiring an integration limit, see 2.4.
Equation (11)f will give:

$$W_{n,elstat} = 4\pi\varepsilon_c\rho_0^2 \int_0^{r_n} \exp(-(\rho_n/r)^3) r^{-2}\ dr = b_0\,\Gamma(+1/3, (\rho_n/r_n)^3)\,\rho_n^{-1}/3 \approx b_0\,\Gamma_{+1/3}\,\rho_n^{-1}/3 \tag{13}$$

Particles are supposed to be electromagnetic objects possessing photon-like properties, thus it will be assumed that particle energy, W_n, has equal contributions of electric and magnetic energy, i.e.:

$$W_n = W_{n,elstat} + W_{n,mag} = 2W_{n,elstat} \approx 2\,b_0\,\Gamma_{+1/3}\,\rho_n^{-1}/3 \tag{14}$$

2.4 Integration limits, spin, parameters in ρ (σ, ß)

2.4.1 Integration limits

Apart from the electrostatic potential according to chpt. 2.2 two additional coefficients will be part of ρ that can be derived from the integration limits obtained by setting a specific value for spin/angular momentum as boundary condition. The integral limits required for integrals of (12) are either a radius r_n for integrals over $\exp(-(\rho_n/r)^3)$ or $(\rho_n/r_n)^3$ for the Euler integrals.

2.4.2 σ, S_Z = 1/2 [ħ]

The integral limit for the Euler integral will be expressed via a constant defined as 8/σ [10]:

$$(\rho_n/r_n)^3 = 8/\sigma \tag{15}$$

whose value may be derived from the condition for the z-component of angular momentum S_z = 1/2 [ħ]. A simple relation with angular momentum S_z for spherical symmetric states will be given by applying a semi-classical approach:

$$S_z = r_2 x\, p(r_1) = r_2 W_n(r_1)/c_0 \equiv 1/2\ [\hbar] \tag{16}$$

Using term $2b_0$ of equ. (14) as constant factor and integrating over a circular path of radius $|r_2| = |r_1|$, equation (12) will give for m = 0:

$$S_z = \int_0^{r_n}\int_0^{2\pi} S_z(r,\varphi)\, d\varphi\, dr = 4\pi\frac{b_0}{c_0}\int_0^{r_n} e^{-\left(\frac{\rho_n}{r}\right)^3} r^{-1} dr = 4\pi\alpha\hbar \int_0^{r_n} e^{-\left(\frac{\rho_n}{r}\right)^3} = \frac{4\pi}{3}\alpha\hbar\int_{8/\sigma_0}^{\infty} t^{-1} e^{-t} dt$$
$$= \frac{4\pi}{3}\alpha\hbar\,\Gamma(0, (\rho_n/r_n)^3) \equiv \frac{[\hbar]}{2} \tag{17}$$

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9 Euler integrals yield positive values, the sign convention of Γ-functions gives negative values for negative arguments. Abbreviations such as $\Gamma_{-1/3}$ for |Γ(-1/3)| will be used;

10 Chosen to give coefficient σ as component in the argument of the exponential of Φ according to [A3.1].

To obtain $S_z = 1/2$ [ħ] the integral over $\exp(-(\rho_n/r)^3)/r\, dr$ of (17), *has to* yield $\alpha^{-1}/8\pi$.

$$\int_0^{r_n} \exp\left(-\left(\rho_n/r\right)^3\right) r^{-1} dr \;=\; 1/3 \int_{8/\sigma_0}^{\infty} t^{-1} e^{-t}\, dt \;\equiv\; \frac{\alpha^{-1}}{8\pi} \approx 5.45 \qquad (18)$$

Relation (18) may be used for a numerical calculation of the integration limit, $8/\sigma_0$ (assumed to represent spherical symmetry), **σ_0 = 1.810E+8 [-]**.

Assuming the coefficient $\Gamma_{-1/3}/3$ according to (12), (25) has to be part of the expression for σ_0 this results in an approximation for σ_0 as:

$$\sigma_0 \approx 8\,(1.524\, \alpha^{-1}\, \Gamma_{-1/3}/3)^3 \approx 8\,(1.5\, \alpha^{-1}\, \Gamma_{-1/3}/3)^3 \qquad (19)$$

The main contribution in form of an α-term has to be expected [11], some details, including the residual factor of 1.525 from the fit (usually approximated as 1.5 in the following), are discussed in [A4.1]).

The minimal possible value for σ, given by the mathematical constants in (19)

$$\sigma_{min} = 8(\Gamma_{-1/3}/3)^3 \qquad (20)$$

leaves a term σ_0/σ_{min} as variable part in σ to consider non-spherical symmetric states, cf. chpt. 2.5.3:

$$\left(\sigma_0/\sigma_{min}\right)^{1/3} = \alpha_{\lim} \approx 1.524\,\alpha^{-1} \approx 1.5\,\alpha^{-1} \qquad (21)$$

2.4.3 Particle “radius”, r

In the following a term for length expressed via the Euler integral of (12) will be introduced for a general length r_x:

$$r_x = \int_0^{r_x} e^{-\left(\frac{\rho_n}{r}\right)^3} dr \;=\; \rho_n/3 \int_{(\rho_n/r_x)^3}^{\infty} t^{-4/3} e^{-t}\, dt \;\approx \Gamma(-1/3, (\rho_n/r_x)^3)\;\rho_n/3 \qquad (22)$$

In the limit $(\rho_n/r_x)^N \to 0$

$$\Gamma(-1/N, (\rho_x/r_x)^N) = \int_{(\rho_x/r_x)^N}^{\infty} t^{-(1/N+1)} e^{-t} dt \;\approx N\,(\rho_x/r_x)^{-1} = N\,\sigma_x^{1/3}/2 \qquad (23)$$

holds (last term using (15)). For the case $\sigma_x = \sigma_0$ and N=3:

$$r_n \approx\; 3/2\; \sigma_0^{1/3}\, \rho_n/3 = \sigma_0^{1/3}/2\; \rho_n \qquad (24)$$

Considering term $2/3\Gamma_{-1/3}$ has to be part of σ_0 would give for the mathematical coefficients involved in r_n of (24)

$$r_x \sim 2/3\; \Gamma_{-1/3}/2 = \Gamma_{-1/3}/3 \qquad (25)$$

as the limit of the Γ-term for length.

The 3rd term of equation (23) inserted in the right side of (22) gives back r_x, however, the relations of (22)f may be seen as expressing r_x in terms useful for this model, i.e. ρ_n, σ_0 and Γ-functions.

The values calculated with relation (23) will be related to the Compton wavelength, $\lambda_{Cn} = hc_0/W_n$, by a factor √3 (using (14) for W_n and (19), (24) for r_n plus (78)) [12]:

$$\lambda_{C,n} = 3\rho_n hc_0/(2b_0\Gamma_{+1/3}) = 3\pi\,\alpha^{-1}\rho_n/\Gamma_{+1/3}; \qquad r_n \approx \sigma_0^{1/3}/2\rho_n \approx \alpha^{-1}\,\Gamma_{-1/3}\,\rho_n/2$$
$$\Rightarrow\quad \lambda_{C,n}/r_n \approx 6\pi/(\Gamma_{+1/3}\Gamma_{-1/3}) = 6\pi/(2\pi\sqrt{3}) = 3^{0.5} \qquad (26)$$

A value for r_e calculated with these equations and σ_0 of 2.4.2 is not in agreement with a numerical calculation of r_e with boundary condition $S_z = 1/2$ [ħ] *unless* σ_0 and ß (for charged particles, as discussed below) are part of the argument of the exponential of Φ [13]:

$$\rho^3 \approx \sigma_0 ß\, \rho_0^3 \qquad (27)$$

i.e. in an expression such as (24) the σ-term has to be considered twice, giving $r_n \sim \sigma^{2/3}$.

2.4.4. Other values for integration limits, S = √3/2 [ħ], S = 1 [ħ]

For any given ρ in the exponential a unique corresponding integration limit, r_n/σ_n is defined via (12). However, there will be other relevant length/σ-parameters that may be defined by alternative boundary

11 In 0th approximation: using the term for energy (14) and length (22)f requires $\sigma^{1/3}$ to be of order of the inverse fine-structure constant α^{-1}: $1/c_0\int w(r)\, dr * \int dr \approx b_0/(c_0\rho_n)\, (\sigma^{1/3}\rho_n) \equiv\; \hbar/2 \;\Rightarrow\; \sigma^{1/3} \approx \alpha^{-1}$.

12 Coefficient √3 of S/S_Z is not reflected in the length parameter r_l since equations such as (17), (22)f are non-linear.

13 A more complex exponential e.g. the discriminant form of [A3.1] would require σ to be part of the exponent as well.

conditions [14].

Next to the boundary condition S_Z = 1/2[ħ], assumed to represent restrictions for spin due to the (average) fields of a particle state, S = √3/2[ħ] will mark an *absolute limit* for a particle state and S = 1[ħ] will mark a characteristic length *beyond* a particle state.

2.4.4.1 ß, S = √3/2 [ħ]

For particles associated with charge (or partial charges) a coefficient in addition to σ will be needed. A rough estimation may be obtained from the condition that energy/length of a charged particle has to be higher/smaller than the value given by a pure electrostatic term. Since r_l according to (19), (24), (27) will be proportional to α^{-2} the term in the exponential has to be: $ß < \alpha^6$ [15].

Coefficient ß may be calculated directly from the condition that at a particular length parameter the terms in the expression for length according to chpt. 2.4.3 will cancel, i.e. the σ/length-terms which are larger than 1 will cancel ß < 1 to give $r_l = \rho_0 = e_c/(4\pi\varepsilon_c)$. This value may be interpreted as transition from a particle state to a point charge state and will be identified with the maximum possible value for a spin state of a fermion, S = √3/2[ħ].

$$r_x = r_e \frac{r_l}{r_e} \approx \left(1.5^3 \sigma_0^2 ß \rho_0^3\right)^{1/3}/2\left(\frac{r_l}{r_e}\right) \approx \rho_0 = r_l \qquad (28)$$

The ratio of r_l/r_e can be taken e.g. from $r_l/r_e = \sqrt{3}e_c/(4\pi\varepsilon_c\lambda_{C,e})$, a numerical calculation for r_l [A9], etc. to give:

$$ß' \approx 8\left(1.5^3\sigma_0^2\right)^{-1}\left(\frac{r_e}{r_l}\right)^3 \approx 8\left(1.5^3\sigma_0^2\right)^{-1}\left(\frac{4\pi\varepsilon_c\lambda_{C,e}}{3^{0.5}e_c}\right)^3 = 4.870E\text{-}22 \qquad (29)$$ [16]

2.4.4.2 S = 1 [ħ]

For the series expansion discussed in the next chapter the approach of 2.4.4.1 gives a sufficient approximation. However, to cancel the minor terms of σ_{lim} the exact relation would require a limit of

$$r_{ll} = 2/3\ \Gamma_{-1/3}\ e_c/(4\pi\varepsilon_c) = 2/3\ \Gamma_{-1/3}\ r_l \qquad (30)$$

and absence of the extra factor of the electron (≈1.5, see 2.5.1f) in the exponential. The corresponding value of spin will be S = 1[ħ], cf. [A9].

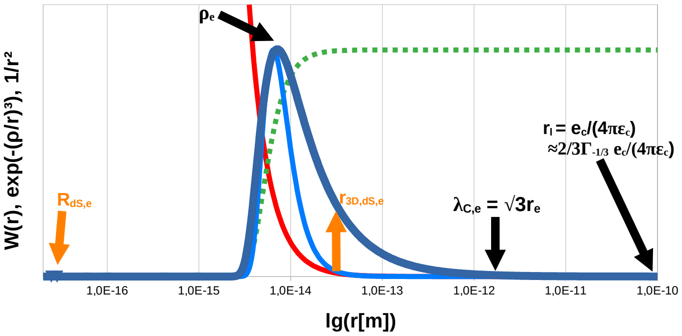

Fig. 1: characteristic lengths in relation to the *electron*; red line: $1/r^2$; green dots: exponential of Φ; dark blue: W(r), light blue: W(r,φ,ϑ); orange: de Sitter radius R_{dS}≙5D, $r_{3D,dS,e}$≙3D, see chpt. 3.3.

2.5 Particle energy

2.5.1 Lower limit of energy

According to (27) of chpt. 2.4.3 σ_0 will be part of the exponential. Calculating energy according to $\rho^3 = \sigma_0\rho_0^3$ and (14) will give W in the order of magnitude of 0.1 eV, a value in the estimated energy range for a neutrino, see tab.1.

Charge will require the additional coefficient ß < 1, as derived in 2.4.4.1. With (14) and (27) one gets for the energy of a ground state W ≈ 1.5 W_e. Factor $(3/2)^3$ in the exponential of Φ or its inverse, 2/3 in the energy expression needed for the electron has up to now no satisfactory explanation [17].

14 For corresponding calculations the original coefficients $\sigma_0ß\rho_0^3$ may be used in numerical calculations as shown in [A9] or in variations of the exponential as e.g. in (70). Alternatively the σ-value may be adapted accordingly.

15 The relationship between a photon-like object and a point charge object involves α, suggesting a photon-like state to differ by a factor of α from a pure point charge state and to use a ground state coefficient $ß_0 \approx \alpha^9$.

16 In the calculations below ß of (38) will be used; ß' = 0.994 ß

17 Factor ~ 1.5 is discussed in [A3.2]. A dependence of interconnected particle sub-states, such as the spin-relation of [A3.5] may allow a deviation in a term that has its boundary not defined by the next sub-state but by free space.

2.5.2 Quantization with powers of $1/3^n$ over α

Most relations given here are valid for any particle energy which should be expected as there is a continuous spectrum of energies according to special relativity. However, a particular set of energies may be identified by relaxing the condition of orthogonality of different states according to quantum mechanics to requiring different states to be expressible in simple terms of a ground state coefficient in the exponent of Φ and not to exhibit any dependence on intermediary states [18].

In chpt. 3.1 a motivation will be given to split ß into a part supposed to represent spherical symmetric states, α^9 times the irregular coefficient of the electron, 1.5^3, and a part representing contributions of non-spherical symmetric states, represented by α_{lim} of (21).

$$\frac{1.5^3\alpha^9}{2\alpha_{\text{lim}}} \approx 1.5^2\alpha^{10}/2 \approx 4.8E\text{-}22 = ß \tag{31}$$

The α^9 part of ß will take the role of the ground state coefficient to give the following partial product:

$$W_n/W_e \approx 3/2\frac{\alpha\wedge(1.5/3^n)}{\alpha^{1.5}} \approx 3/2\Pi_{k=1}^{n}\alpha\wedge(\text{-}3/3^k) = 3/2\,\alpha(n) \qquad n = \{1;2;..\} \quad [19] \tag{32}$$

With (14) this gives (term for ß in bold, 1.5^δ = extra coefficient for the electron only):

$$W_n \approx 2\, b_0\, \Gamma_{+1/3}/3\, \rho_n^{-1} \approx 2\, b_0\, \Gamma_{+1/3}/3\, [(1.5^3)^\delta\, \sigma_0\, \boldsymbol{\alpha_{lim}^{-1}/2\ 1.5^3\,\alpha^9}\, \alpha^{4.5}/\alpha\wedge(4.5/3^n)\ \ (e_c/(4\pi\varepsilon_c))^3]^{-1} \quad n = \{1;2;..\} \tag{33}$$

2.5.3 Non-spherical symmetric states

Assuming the angular part to be related to spherical harmonics and exhibiting the corresponding nodes would give the analog of an atomic p-state for the 1st angular state, y_1^0. With the additional assumption that $W_{n,l} \sim 1/r_{n,l} \sim 1/V_{n,l}^{1/3} \sim (2l+1)^{1/3}$ ($V_{n,l}$ = volume) is applicable for non-spherically symmetric states this would give $W_1^0/W_0^0 = 3^{1/3} = 1.44$ and $\sigma^{1/3} = 3^{-1/3}\,\sigma_0^{1/3}$ [20]. The quaternion model of chpt. 4.1 supports that a y_1^0-like symmetry for particle states has to be considered and a second partial product series of energies in addition to (32) corresponding to these values approximately fits the data, see tab. 1 [21].

A change in angular momentum has to be expected for a transition from spherical symmetric states, y_0^0, to y_1^0 which is actually observed with ΔJ = ± 1 except for the pair μ/π with Δ J = 1/2.

With $\sigma_{ylm}^{1/3} = \{2\Gamma_{-1/3}/3\ 1.5/\alpha$ (for y_0^0); $3^{-1/3}2\Gamma_{-1/3}/3\ 1.5/\alpha$ (for y_1^0); $2\Gamma_{-1/3}/3$ (for maximum)} energy relative to the electron state may be given as:

$$W_n/W_e \approx 3/2\frac{\alpha\wedge(1.5/3^n)}{\alpha^{1.5}}\frac{\sigma_0^{1/3}}{\sigma_{ylm}^{1/3}} = 3/2\Pi_{k=1}^{n}\alpha\wedge(\text{-}3/3^k)\ \frac{\sigma_0^{1/3}}{\sigma_{ylm}^{1/3}} \qquad n = \{1;2;..\} \tag{34}$$

According to α_{lim} the maximum additional contribution relative to a spherical symmetric state would be:

$$\Delta W_{max} \approx 3/2\ \alpha^{-1}\ . \tag{35}$$

resulting in a total maximum of energy of $W_{max,total} \approx W_e\ 9/4\ \alpha^{-2.5}$ = 4.05E-8 [J] (= 1.028 Higgs vacuum expectation value, VEV = 246GeV = 3.941E-8 [J] [7]) [22].

2.5.4 Results of energy calculation

Table 1 presents the results of the energy calculation according to (14), (33) for y_0^0 (bold), y_1^0. Only states given in [7] as 4-star, characterized as *„Existence certain, properties at least fairly well explored"*, are included, up to Σ'^0 all such states given in [7] are listed. Coefficients given in col. 4 refer to (32)ff starting with the electron coefficient in W_e, including its extra term set exactly to 2/3. Exponents of -9/2 for Δ and tau are equal to the limit of the partial product of α(n), including the electron coefficient.

Col. 5 gives energy calculated with σ_0 according to the value of the fit for S_Z =1/2 of 2.4.2 and ß given by W_e/W_{Pl}, (38), according to the experimental value of the electron and definition (36) for Planck energy.

18 cf. $W_n^2 \sim \left(\alpha_0^{1/3}\alpha_0^{1/9}....\alpha_0^{1/(3\wedge(n\text{-}1))}\boldsymbol{\alpha_0^{1/(3\wedge n)}}\right) / \left(\boldsymbol{\alpha_0^1}\alpha_0^{1/3}\alpha_0^{1/9}....\alpha_0^{1/(3\wedge(n\text{-}1))}\right) = \boldsymbol{\alpha_0^{1/(3\wedge n)}/\alpha_0}$, cf. [A3.5] as well;

19 A supposed neutrino state according to 2.5.1 would roughly fit such a 3rd power partial product as well: $W_n/W_\nu \approx \Pi^n_{k=0}\ \alpha\wedge(\text{-}3/3^k)$ n={0;1;..}.

20 Allocating all aspects related to angular momentum to some σ-term;

21 Considerations such as given in [A7] may give some indication why a 1:1 correspondence between y_1^0-like states and mesons should not be expected.

22 Factor 1.042 with σ_0 of 2.4.2 and according to (33). For the Higgs boson see [A7.3].

	n, l	$W_{n,Lit}$ [MeV]	α-coefficient in W_n $\alpha(n)^{-1/3}$ $[\sigma_o / \sigma_{ylm}]^{1/3}$	W_{calc}/W_{lit}	J
ν	**-1***	**'~ E-7**	**0**	-	-
e^{+-}	**0, 0**	**0.51**	**$2/3\ \alpha^{-3}$**	**1.014**	**1/2**
μ^{+-}	**1, 0**	**105.66**	**$\alpha^{-3}\alpha^{-1}$**	**1.008**	**1/2**
π^{+-}	1, 1	139.57	$\alpha^{-3}\alpha^{-1}$ $[3^{1/3}]$	1.101	0
K		495	[A7]		0
η^0	**2, 0**	**547.86**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}$**	**0.991**	**0**
ρ^0	2, 1	775.26	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3})$ $[3^{1/3}]$	1.022	1
ω^0	2, 1	782.65	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3})$ $[3^{1/3}]$	1.012	1
K*		894	[A7]		1
p^{+-}	**3, 0**	**938.27**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**1.011**	**1/2**
n	**3, 0**	**939.57**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**1.010**	**1/2**
η'		958	[A7]		0
Φ^0		1019	[A7]		1
Λ^0	**4, 0**	**1115.68**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}$**	**1.020**	**1/2**
Σ^0	**5, 0**	**1192.62**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81}$**	**1.014**	**1/2**
Δ	**∞, 0**	**1232.00**	**$\alpha^{-9/2}$**	**1.012**	**3/2**
Ξ		1318			1/2
Σ^{*0}	3, 1	1383.70	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9})$ $[3^{1/3}]$	0.989	3/2
Ω^-	4, 1	1672.45	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27})$ $[3^{1/3}]$	0.982	3/2
N(1720)	5, 1	1720.00	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81})$ $[3^{1/3}]$	1.014	3/2
tau^{+-}	∞, 1	1776.82	$(\alpha^{-9/2})$ $[3^{1/3}]$	1.012	1/2
Higgs	∞,∞ **	1.25 E+5	$(\alpha^{-9/2})$ $[3/2\ \alpha^{-1}]\ /2$	1.022	0
VEV	∞,∞**	2.46 E+5	$(\alpha^{-9/2})$ $[3/2\ \alpha^{-1}]$	1.042	

Table 1: Particle energies; col.2: radial, angular quantum number; col.4: α-coefficient in W_n according to (32)f, however, with n = {0;1;2;..} to include electron coefficient $2/3\alpha^{-3}$; col.5: ratio of calculated energy, W_{calc} according to (33)f and literature value [7] (* see 2.5.1, ** chpt. 2.5.3, [A7.3]); col.6: angular momentum S_z [ħ];

Additional particle states and blanks in the table are discussed in [A7]. The values of physical constants are taken from [7].

To illustrate possible QED-effects and the non-linearity of the Γ-functions: a calculation of σ_0 with values of (17)f varying within +/-1.00116 gives a range of energy values of +/-1.006, varying within $+/-1.00116^2$ gives a range of energy values of +/-1.013 compared to the values given in table 1 [23]. Additional effects due to e.g. different charge in particle pairs of same isospin have to be expected.

The accuracy of ~1% of the values calculated for leptons, mesons and baryons is comparable to that of LQCD calculations for baryons [8].

3 Higher order phenomena - gravitation, de Sitter space

3.1 Planck scale

In this work the expression

$$b_0 = G\, m_{Pl}^2 = G\, W_{Pl}^2 / c_0^4 \qquad (36)$$

is used as definition for Planck terms, giving for the Planck energy, W_{Pl}:

$$W_{Pl} = c_0^2\, (b_0 / G)^{0.5} = c_0^2\, (\alpha\hbar c_0 / G)^{0.5} = 1.671\ E{+}8\ [J] \qquad (37)$$

The value of W_{Pl} according to definition (37) allows to identify the ratio of W_e and W_{Pl} with the α-terms given in (31), i.e. the relation between W_e and W_{Pl} is given by $\alpha_e \approx (3/2)^3\, \alpha^9$, the electron coefficient in the

23 This involves Γ(0,x); the nonlinearity of Γ-functions in the parameter range of this model increases Γ(+1/3,x) << Γ(0,x) << Γ(-1/3,x), see fig. 5 in [A4.1].

exponential part of Φ, divided by two times the factor α_{lim} for non-spherical symmetric contributions according to (21). For the calculations in this work ß = W_e/W_{Pl} will be used.

$$\frac{1.5^3\alpha^9}{2\alpha_{\lim}} \approx 1.5^2\alpha^{10}/2 \approx 4.8E\text{-}22 \approx ß \equiv \frac{W_e}{W_{Pl}} = 4.8996E\text{-}22 \quad (38)$$

The constant G may be given as:

$$G \approx \frac{ß^2 c_0^4 b_0}{W_e^2} \quad (39)$$

Since ß and W_e may be expressed as function of π, $\Gamma_{+1/3}$, $\Gamma_{-1/3}$ and e_c, (14), (33), (38) and (79), G may be expressed as a coefficient based on EM constants only, $G \approx 2/3\ c_0^4\alpha^{24}/(4\pi\varepsilon_c) = 6.6799E\text{-}11\ [m^3/(kgs^2)]$.

3.2 Gravitation as second order term of series expansion of exponential function

Terms for gravitation may be recovered via a series expansion of either $\Gamma(+1/3, (\rho_n/r_n)^3)$ [24] of (13) or the exponential part of Φ in any suitable expression, e.g. potential $e_c/(4\pi\varepsilon_c r)$, resulting in a general term such as:

$$\frac{e_c}{4\pi\varepsilon_c r}\left[1 - \sigma ß\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right] \approx \text{Coulomb-term}\left[1 - \sigma ß\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right] \quad (40)$$

which is a very good approximation for r > $\alpha\lambda_C$. The 1st term is the classic Coulomb term, the 2nd term contains by definition, equ. (38), the ratio between Coulomb and gravitational terms for *one* electron, ß. To turn this into the exact Coulomb / gravitation relationship requires:

1) parameter r in $e_c/(4\pi\varepsilon_c r)$ to turn into a constant,
2) coefficient σ to approach σ_{min}
3) parameter r to approach the value $r_{ll}= 2/3\ \Gamma_{-1/3}\ e_c/(4\pi\varepsilon_c)$, the product of $\sigma_{min}^{1/3}$ and r_l.

=>1) r *in the exponential* may not be a free parameter for $r \geq r_l \approx r_{ll}$ the radius value that marks the maximal possible value for spin of a fermion and the approximate limit of a real solution for an equation such as (70),

=> 2), 3) coefficient σ is essentially related to spin of a particle and it has to be assumed that spin does not play a role for r > r_l. Specifically this corresponds to the limit $\sigma_{min} = (2\Gamma_{-1/3}/3)^3$. An equivalent term in r will exist for the boundary condition S = 1 [ħ], which will erase the special factor ≈ 3/2 of the electron as well (cf. chpt. 2.4.4.2, [A9]), leaving ß as only remaining coefficient.

$$\frac{2\Gamma_{-1/3}}{3}\left(\frac{e_c}{4\pi\varepsilon_c}\right)\left(\frac{2\Gamma_{-1/3}}{3}\ r_l\right)^{-1} = 1 \quad (41)$$

The general expression for the series expansion will be:

Coulomb-term (1 - ß + ß²/2 - ....) (42)

Coefficient ß², necessary to give the full equivalent for replacing the constant G, is evenly split on both particles involved in gravitational interaction, i.e. the second ß has to be contributed from the second particle, multiplied by appropriate coefficients from the α-series according to the α(n) of (32)f and σ coefficients of 2.5.3 for particles other than the electron (in their rest frame). Since the 2nd term of such a series expansion should not exceed the 1st, electromagnetic one, the maximum (relativistic) mass for spherical symmetric particles would be defined by $\alpha_e^{-1}= ((3/2)^3\ \alpha^9)^{-1}$ while the maximum angular term, α_{lim} as given in (21), secures that particles that are not spherical symmetric in a rest frame can not exceed the Planck limit either. Restricting to electrostatic contributions only [25] will give (38).
The force of gravitation between two particles 1 and 2 would be given by:

$$F_G = \frac{G m_1 m_2}{r^2} = \alpha(1)\alpha(2)ß^2 F_C = \frac{\alpha(1)\alpha(2)ß^2 e_c^2}{4\pi\varepsilon_c r} = \frac{\alpha(1)\alpha(2)\gamma_\#^2 W_e^2}{4\pi\varepsilon_c r} \quad [26] \quad (43)$$

The maximum general, i.e. non rest-frame coefficient allowed for a particle in a symmetric case would be $ß^{-1}$.

---

24 $\Gamma(1/3, (\rho_n/r)^3) \approx \Gamma_{1/3} - 3(\rho_n/r) + 3/4(\rho_n/r)^4 - 3/7(\rho_n/r)^7 + ...$ [9]

25 I.e. factor 2 in the denominator might correspond to relate only the electrostatic contributions of (14) for the electron with the electrostatically defined value of a Planck state, equ.(36).

26 In the last term energy refers to W_e in place of e_c, ß² has to be replaced by $\gamma_\#^2$ of (9).

In equations the coefficients ß (1 particle, potential) and $ß^2$ (2 particles, interaction) will indicate the transition from EM to gravitational scale, i.e. these coefficients replace a G-term, the 3rd term in the series expansion should feature $ß^2$ and $ß^4$.

3.3 The 3rd order term of the series expansion

The following will examine some links between Λ / deSitter-space / MOND, based mainly on ideas of Milgrom [11] and Aldrovandi, Pereira, et al. [12], and the results of this modified Kaluza ansatz, in particular the series expansion of chpt. 3.2.

3.3.1 Λ / de Sitter-space / MOND / BTFR

The proportionality constant in the Einstein Field Equations (EFE), $8\pi G/c_0^4$, originates from equating $G_{00} = T_{00}$ in the "weak field limit" (WFL) of low acceleration with the corresponding term from Newton's law. With a non-zero Λ this relation can not be strictly valid any more and a deviation from the Newton case due to Λ has to be expected [27].

MOND (Modified Newton Dynamics) is an ansatz for the WFL that attempts to interpret a characteristic acceleration, $a_0 \approx 1.2\text{E-}10$ [m/s], observable in several astrophysical phenomena such as the Baryonic-Tully-Fisher-Relation (BTFR). Early on a numerical relationship between a_0 and the cosmological constant, Λ_0, as well as a possible relationship with a 4D-de Sitter space (dS_4) has been pointed out [11].

De Sitter space is the maximally symmetric solution of the EFE with a positive constant, Λ. It is characterized by a constant energy density and can be described as a submanifold of a Minkowski space of dimension n+1, for dS_4:

$$\pm x_0^2 + x_1^2 + x_2^2 + x_3^2 + x_4^2 = R_{dS}^2 \qquad (44)$$

R_{dS} is a length parameter, the "radius" of the de Sitter space. The scalar curvature R of de Sitter space is given by

$$R = \frac{n(n-1)}{R_{dS}^2} = \frac{2n}{n-2}\Lambda . \quad \text{(n =dimension of embedded space)} \qquad (45)$$

for dS_4: $\Lambda = 3/R_{dS}^2$. This corresponds to an embedding in a flat 5D-space-time, the general approach on which this modified Kaluza model is based as well.

+/- x_0 represents a time coordinate, the sign defines hyperbolic or spherical symmetry of the dS_4. In the following only symmetry of a 3-sphere for space and great circles as geodesics will be considered, i.e. a closed solution (or imaginary time e.g. in the case of particles [28]) will be assumed.

In such a dS_4-geometry any worldline would formally have an acceleration component a_0*:

$$a_0^* = v^2/R_{dS} = c_0^2(\Lambda_0/3)^{0.5} \approx 5.46\text{E-}10[\mathrm{m/s^2}] \approx 4.55\, a_0 \quad {}^{29} \qquad (46)$$

factor 4.55 has a geometric interpretation, see (52).

One of the best-documented deviations from Newton's limit of GR in the WFL, as expected according to baryonic mass, is represented by the BTFR, the strong correlation between total baryonic mass (stars and gas) in a galaxy and rotation speed that holds in a wide range of distances far from the galactic center for various types of galaxies [14]. It is characterized by a forth-power-relation of rotation velocity, v_f = "final" rotation velocity in the WFL, with the total baryonic mass of the galaxy, M_G:

$$a_0 G \mathrm{M_G} = v_f^4 \qquad (47)$$

This corresponds to the so called deep-MOND regime where $a \ll a_0$. In relation to GM_G the equations (46) and (47) have a_0 in a reciprocal relationship, the dS_4 interpretation does not fit to MOND in a trivial way.

3.3.2 Λ / de Sitter-space / MOND / BTFR in the modified Kaluza model

If the second term in the series expansion of (42) represents gravitation, it is expectable that the third term represents deviations from gravitation.

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27 In a simple ansatz the effects of Λ on gravitational phenomena would be relevant on the scale of intergalactic interaction and Mpc [13]. Araujo et al. consider a local Λ-term as function of energy density, Λ(r) ~ w(r), in "de Sitter modified general relativity" to describe the BTFR [12b].

28 Imaginary time might be appropriate for the particles of this model since the exponential function of the t-coordinate responsible for the expansion of a hyperbolic dS_4 will turn into a periodic function in a spherical dS_4.

29 Λ_0 according to [16].

3.3.2.1 Λ and a_0

For assigning a value to R_{dS} additional assumptions will be needed. A simple ansatz might be:

$$\frac{3\,ß^4}{R_{dS,e}^2} \equiv \Lambda_0 \tag{48}$$[30]

with $R_{dS,e}$ = 3.954E-17[m] tuned to give a simple relationship between $ß^4$ and Λ_0. Coefficient ß may be multiplied with appropriate α(n)-terms of (34) for particles other than the electron or ratios of the relevant mass to the electron mass [31].

The link between R_{dS} and a corresponding r of a particle in 3D-space will be established via energy density, cf. [8.2]:

$$w = \frac{4\pi\varepsilon_c}{R_{dS,e}^2} = 4\pi\varepsilon_c\left(\frac{e_c}{4\pi\varepsilon_c r_{x,e}^2}\right)^2 \tag{49}$$

giving for $R_{dS,e}$ of (48) a value of $r_{x,e}$ ≈ 5.42E-14 [m] that is in a reasonable range for an electron, see fig.1 [32].

In the following (48) will be used as input together with the additional assumption that any relevant parameter of dimension length, including term $e_c/(4\pi\varepsilon_c)$, has to be assigned the length parameter $2/3\Gamma_{-1/3}$ as given in (41).

I.e. $r_{x,e}^4$, that can be decomposed according to

$$\left(\frac{e_c}{4\pi\varepsilon_c r_{x,e}^2}\right)^2 = \left(\frac{e_c}{4\pi\varepsilon_c r_l}\right)^2 \frac{1}{R_{dS,e}^2} = \frac{1}{R_{dS,e}^2} \tag{50}$$

into r_l, that cancels the electromagnetic term, and R_{dS}, will be interpreted as

$$\left(\frac{e_c}{4\pi\varepsilon_c r_{x,e}^2}\right)^2 = \left(\frac{2\Gamma_{-1/3}}{3}\right)^2\left(\frac{2\Gamma_{-1/3}}{3}\right)^{-4}\left(\frac{e_c}{4\pi\varepsilon_c r_{x,e}^2}\right)^2 = \left(\frac{2\Gamma_{-1/3}}{3}\right)^{-2}\frac{1}{R_{dS,e}^2} \tag{51}$$

This allows to identify factor 4.55 in (46) as:

$$a_0{*}^2 = c_0^4(\Lambda_0/3) \approx \left(5.46\text{E-}10\,[\mathrm{m/s^2}]\right)^2 \approx 4.55^2 a_0^2 \approx 3\left(\frac{2\Gamma_{-1/3}}{3}\right)^2 a_0^2 \tag{52}$$

The $(3^{0.5})^2$ signifies that $3/R_{dS}^2 = \Lambda_0$ might be an appropriate reference for acceleration rather than $1/R_{dS}^2 = \Lambda_0/3$. Constructing a 3rd order term with (42) that is equivalent to Gm_e will require to replace e_c in the denominator by W_e to give:

$$\frac{G\,m_e}{r^2} = \frac{ß^4 c_0^4}{r^2}\,\frac{e_c^2}{ß^2 c_0^2 W_e 4\pi\varepsilon_c}\,\frac{1}{2} \tag{53}$$

Using $R_{dS,e}$ according to (48) and relation (51)f as input in (53), i.e. assuming $a_0{*}^2$ (blue) to be a part of expression (53) gives:

$$\frac{G\,m_e}{R_{dS,e}^2(2\Gamma_{-1/3}/3)^2} = \frac{ß^4 c_0^4}{R_{dS,e}^2(2\Gamma_{-1/3}/3)^2}\,\frac{e_c^2}{ß^2 c_0^2 W_e 4\pi\varepsilon_c}\,\frac{1}{2} \approx \frac{a_0^{*\,2}}{a_{0,calc}} \tag{54}$$[33]

The additional $(2\Gamma_{-1/3}/3)^2$-term turns the remaining parts of (54) into a good approximation of a_0 (red) [34]. The blue part essentially depends on the choice of the value of R_{dS} and leaves some room for speculation. Using (48) might be a rather unbiased choice, turning it into a_0^2 might be another, either by adapting R_{dS} or expanding with $3(2\Gamma_{-1/3}/3)^2$. Anyway, with any electron-related version of (49) the nominator of (54) will be a term in the rough order of a_0^2 while the denominator will be a_0 in any case. A more fundamental metric will

30 Coefficient 3/2 will not be part of the equations due to the limit r_{ll}, cf. chpt. 2.4.4.2.

31 The maximum insertable coefficient of $ß^{-4}$ would give back the bare cosmological constant, the corresponding energy would be $ß^{-4}W_e$ ≈ 1.4E+72J, roughly in the range of the energy of the observable universe, which itself is dark energy dominated and approaching a de Sitter state.

32 Any other choice for $r_{x,e}$, in the range of electron length parameters $r_{max} < r_{x,e} < \lambda_{C,e}$ would still yield values close in order of magnitude to Λ_0 (as would direct terms from the metric, see [A8.1]).

33 Inserting (50) in (54) gives terms of $(e_c/(4\pi\varepsilon_c))^3$, corresponding to terms that might be derived directly from an appropriate metric, cf. [A8.1].

34 $a_{0,\,calc}$ = 1.123E-10[m/s^2] = 0.936*1.2E-10[m/s^2];

have to clarify to what extent (48) is an appropriate ansatz. Still equ. (54) demonstrates that the specific terms of this model can be rearranged to approximately reproduce both the a_0-Λ_0-relation of (46) and the BTFR-relation of (47).

3.3.2.2 Baryonic-Tully-Fisher-Relation (BTFR)

The expression for the BTFR, (47), is not dependent on r, thus assumptions about R_{dS} or r_x as in chpt. 3.3.2.1 are not necessary. Solving (54) to give $a_0 G m_e$ results in the "BTFR" for 1 electron. To get to the galaxy scale this has to be multiplied by M_G divided by the mass of the electron [35], giving for v_f^4 (M_{sol} ≙ solar masses [15]):

$$v_f^4 = ß^4 c_0^4 \left(\frac{2\Gamma_{-1/3}}{3}\right)^2 M_G / M_e = 7.50\text{E+9} * M_G / M_{sol} \qquad (55)$$

For an M_G in the order of magnitude of the Milky Way, ≈1E+12 M_{sol} this yields a value of $v_{f,calc}$ ≈ 300km/s (=0.83 $a_0 GM_G$), cf. fig. 2.

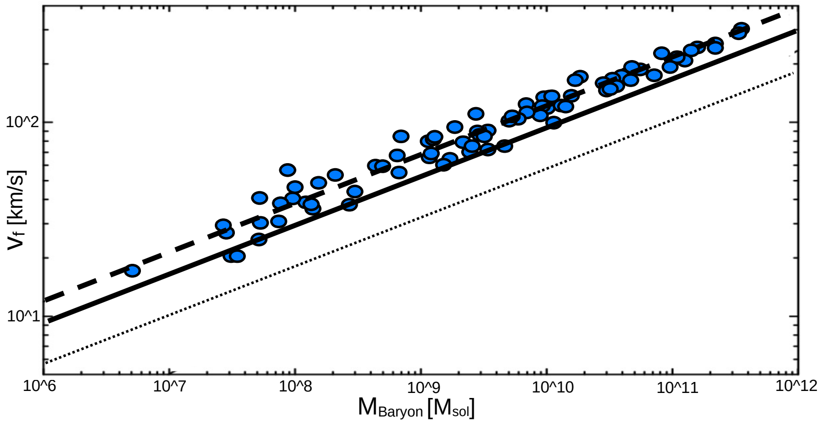

Fig. 2: BTFR, data from fig. 3 of [14a]: blue circles observed values; continuous line of slope 1/4 calculated with (55), dashed line: additional factor of 2 in (55); dotted line: $v_f^4 = ß^4 c_0^4 M_G/M_e$;

Calculating R_{dS} for the Milky Way (≈1E+12 M_{sol}) with (48) gives $R_{dS} = (3ß^4/\Lambda_0\ M_{MW}/M_e)^{0.5}$ ≈ 6E+19[m] ≈ 6[kLy] as an approximate order of magnitude for a transfer to deep-MOND behaviour in a galaxy.

3.3.3 Summary Λ / de Sitter-space / MOND / BTFR

- a non-zero Λ in the EFE requires a deviation from the Newton limit,
- the value of Λ does not fit phenomena such as the BTFR => Λ should be variable => Λ(r), with Λ_0 as limit; this does not contradict the 2nd Bianchi identity since Λ(r), including its lower limit Λ_0, will be a part of $G_{\alpha ß}$, i.e. it has its origin in the metric,
- the numerical relationship $a_0 \approx c_0^2/\Lambda_0^{\ 0.5}$ hints at an interpretation in terms of a dS_4,
- the first 3 terms in the expansion of the exponential of Φ according to (42) may be interpreted as: electromagnetic + gravitational + dS_4-background (Λ, BTFR, etc.), i.e. the dS_4-space is an integral part of the mathematical framework of this model,
- the dS_4 is centered at the center of the potential well, mass/energy distribution,
- trajectories on great circles in the WFL might be based on the geometry of a dS_4,
- a quantitative link between R_{dS} and a 3D-radius may be given via energy density according to (49)ff,
- the relevant values of Λ_0 and a_0 fit to an origin in parameters of the electron,
- the BTFR can be reproduced qualitatively and quantitatively.

One of the major open questions that remain is the role of Λ_0 as a parameter related to expansion of space-time. As Λ(r) would in general be related to particles this requires to reinterpret the concept of expansion of the universe fundamentally. In (42) the 2nd and 3rd terms have an appropriate opposite sign corresponding to repulsion of the latter. (The analog of the repulsive term (32) in [12b].) The close relation to particles might suggest that expansion on the scale of the universe might correspond to the composite effect of the expansion of the fields of all particles. An exponential of type (70) implies imaginary solutions and a change of sign for the r-coordinate in the metric for $r < r_l$ <=> $r > r_l$, presumably switching from exp(ix) for periodicity / particle state to exp(x) for expansion / external fields.

35 If one prefers the nucleons as a more natural starting point this will require a multiplication by α(n) = 1836 and the number of nucleons for the particular galaxy in (55).

4 Quaternion ansatz

4.1 Basic approach

The model as described above emphasizes a Kaluza-like ansatz with spin as boundary condition. Reversing the main focus, emphasizing angular momentum and implicitly assuming curvature of space as necessary boundary condition for localization is a straight forward alternate way to get additional information about the states of this model, details are given in [A5].

A circular polarized photon with its intrinsic angular momentum interpreted as having its E- and B-vectors rotating around a central axis of propagation, C, will be transformed into an object of SO(3)-type symmetry where the center of rotation is the origin of a triple of EBC-vectors, supposed to be locally orthogonal [36]. This has the following qualitative consequences:

1) Such a rotation is related to the group *SO(3)* and *SU(2)* as important special case. In the following a quaternion ansatz will be used for modelling the respective rotations.
2) E-vector constantly oriented to a fixed point implies *charge*. As implicitly assumed above, neutral particles are supposed to exhibit nodes separating corresponding equal volume elements of reversed E-vector orientation and opposite polarity.
3) A local coordinate system = rest frame implies *mass*.
4) In case of any lateral extension of the E-field, for r -> 0 the overlap of a rotating E-vector implies rising energy density, resulting in *rising curvature of space-time* according to GR or its modification as of equ. (8).
5) The EBC-triple can be given in 2 different *chiral* states (left- right-handed).
6) As essentially electromagnetic waves such states are consistent with a "point-like" structure function on the other hand imply a spatial distribution of energy density and angular momentum / spin.
7) Antiparticles may be constructed by switching orientation of fields and chirality.

For quantitative results 3 orthonormal vectors E, B, C, each described as imaginary part of a quaternion with real part 0, will be subject to alternate, incremental rotations around the axes E, B and C. In the following only solutions where one of the incremental angles of rotation has half the value of the other two will be considered. This may serve as a primitive model for spin $S_Z = 1/2$. There are 3 possible solutions corresponding to half the angular velocity for each of the components E, B, or C. The trajectory of the E-vector encloses a spherical cone, the spherical cap of the cone encompasses a fraction of the area of a hemisphere of 2/3, 1/3 and 1/3, respectively. Mirroring at the center of rotation gives the equivalent double cone (dark grey in fig. 3), the fractions of both caps in relation to the surface of the total sphere may be interpreted to give partial charges of 2/3, 1/3 and 1/3 according to Gauss' law. In the following such components will be assigned to uds-quark-like entities, the assignment (half-frequency-E-rotation, charge +2/3, U), (half-B, charge -1/3, D), (half-C, charge -1/3, S) will be used.

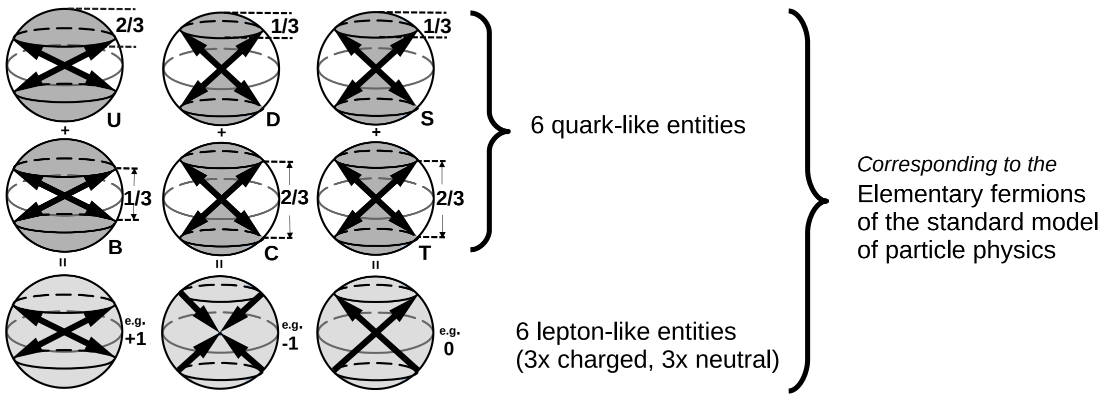

Fig.3: Trajectories of the E-vector, enclosing spherical cones and spherical wedges

The E-vector might as well be interpreted to enclose the complement of the double cone of a 3D-ball (white in fig. 3), a spherical wedge. This gives the objects complement-U, complement-D, complement-S with charges 1/3, 2/3, 2/3. These objects may be assigned to cbt-quark-like entities.
A combination of two cones to give a double cone will always give a valid solution with any spin or chirality

36 In the limit r -> λ_C => |C| -> $|c_0|$; In place of C an equivalent of a Poynting vector may be used; Since in this model the 5th coordinate, x_5, seems to be related to the inverse of the electromagnetic fields some description inspired by a Poynting vector might be a useful approach for x_5.

and may be considered to correspond to the y_1^0 solutions of chpt. 2.6 [37].
The simplest combination of 2 entities (grey + white) of fig. 3 will consist of 2 complementary segments of same charge, chirality, phase, etc., to recover a simple sphere with *no nodal planes* (last row of fig. 3). Such particles should represent the lowest possible energy state, *S_Z = 1/2 should still be valid* and charge could have values of +/-1 or 0. An electron might be considered e.g. as an (anti-U + (U-Complement = B)) particle, however, unlike a B-meson with spin 1/2. While this is not possible with quarks, i.e. objects with particle character, it would represent the simplest solution for such a type of an electromagnetic wave.
The neutral configuration will have to be distinct from all other particles by representing a state where the center of rotation is not at the "tip" of an E-vector, but at its "center", see last row right in fig. 3. This will be an intrinsically neutral particle unlike particles consisting of components of opposite charge, such as the neutron, and a unique solution that for geometric reasons is not suited as component to build other particles. It will not be subject to the conditions related to "charge" and ß as discussed in 2.4 or 2.5.

4.2 Magnetic moments of baryons

A particular sensitive test for such a toy model will be the calculation of the magnetic moments of baryons from an orthogonal combination of 3 U,D,S-units (cf.[A5.2]). It is possible to give values for all combinations of the uds-octet of spin 1/2 that match the experiment within a few percent, however, they have to be selected from a larger set of solutions. Unique solutions require additional boundary conditions, for nucleons this will be isospin: exchanging U- and D-components results in switching the values for magnetic moment of p and n, (cf.[A5.3]).

	|M|Exp[Am²]	|M|Calc[Am²]	|M|Calc/ |M|Exp
p^{+-}	1.41E-26	1.39E-26	0.988
n	9.66E-27	9.55E-27	0.988
p^+/n			1.001187

Table 2: Magnetic moments for proton and neutron (units in standard SI, cf. [A5.2]); |M|exp: [7];

A simple analysis for particles with S-components is not possible due to differences in symmetry (cf. tab. 4 in [A5]) that prevent a simple cancelling of S_z=1/2 components by exchange of U-S- or D-S-units.

4.3 Chirality / "Color"

The orthonormal EBC-vectors feature two possible chiral configurations, right-handed "R" and left-handed "L", suggesting to be a possible source for a factor 3 frequently appearing in the quantitative interpretation of processes involving a quark-antiquark-pair, such as in the decay of the W- or Z-boson or in the coefficient R of electron-positron-annihilation. While this is attributed to the 3 "colors" of quarks in the SM, the same factor would result for any pair of quark-like states having the possibility to exist in triplet-like states, "LL", "RR" and (1/√2) (LR+RL) [38] (referring to an axial vector representing the EBC-configuration).

4.4 Coupling constants

The reasoning of chpt. 2 is based more on a point charge picture, of chpt. 4 on a photon one. The combination of both involves the fine-structure constant, which may be calculated by equating the expressions for point charge and photon energy:

$$W_{Phot,n} = hc_0/\lambda_{C,n} = hc_0 \ / \int^{\lambda_{C,n}} e^{-\left(\frac{\rho_n}{r}\right)^3} dr = W_{n,pc} = 4\pi\varepsilon_c\rho_0^2 \int_0^{r_n} \exp\left(-(\rho_n/r)^3\right) r^{-2}\ dr \tag{56}$$

Equations such as (11)ff are based on the integral over a 3-dimensional point charge term modified by the exponential term according to (5) with N = 3, and a complementary integral - in 3D for length, λ_C - to yield a dimensionless constant. This may be generalized to N dimensions (N ={3; 4}), to give a point charge term (S_N = geometric factor for N-dimensional surface, in case of 3D: 4π; 4D: $2\pi^2$):

37 Composite objects - in particular if composed of 3 UDS-components – may feature sufficient spherical symmetry to approximately conform to the respective symmetry in the energy equation (34). The spherical symmetry of nucleons as assumed in chpt. 2 may be given by a suitable linear combinations of the states discussed in [A5], [A6], cf. [A7.2, η].
38 With a singlet state corresponding to destructive interference;

$$\int_0^r \exp(-(\rho/r)^N) r^{-2(N-1)} \, d^N r \; = \; S_N \int_0^r \exp(-(\rho/r)^N) \, r^{-(N-1)} dr \qquad (57)$$

that has to be multiplied by a complementary integral

$$\int_0^r \exp(-(\rho_n/r)^N) \, r^{(N-3)} dr \qquad (58)$$

Both electroweak coupling constants can be given in 1st approximation as

$$\alpha_N^{-1} \; = \; S_N \; \frac{\Gamma(+m/N)\Gamma(-m/N)}{m^2} \; = \; S_N \frac{\Gamma(+(N-2)/N)\Gamma(-(N-2)/N)}{(N-2)^2} \qquad \text{(m = N-2, cf. (12))} \qquad (59)$$

i.e. as product of the constants in the integrals (57)f, S_N and the Γ-functions. The exact result depends mainly on the integration limit of the second integral, cf. [A4]. For the fine-structure constant this gives $\alpha^{-1} \approx 4\pi \, \Gamma_{+1/3}\Gamma_{-1/3}$. Table 3 summarizes some results, details are given in [A4].

Constant	Calculated values of *inverse* of coupling constant, α_N^{-1}, weak mixing angle		Exp. values
α_{weak}	$2\pi^2 \Gamma_{+1/2} \Gamma_{-1/2} /4 \; = \; \pi^3 \; =$	31.0	30.4-31.7
α	$4\pi \, \Gamma_{+1/3} \Gamma_{-1/3} \; = 4\pi \, \Gamma_{+1/3} \Gamma_{-1/3} =$	136.8	137.036
$\sin^2(\theta_W)$	α/α_{weak}	0.227	0.222-0.231
$\cos(\theta_W)$	m(W-boson)/m(Z-boson)	0.879	0.882

Table 3: Results for 1st approximation of electroweak coefficients [39]

6 Summary

Theory of everything is a somewhat ironic and pompous term and maybe an unachievable goal. At the time Theodor Kaluza's unification of general relativity and electromagnetism was conceived, it came pretty close, yet the emerging theory of quantum mechanics (QM) moved the finish line. It is a common thought ever since that the theory of General Relativity (GR) somehow has to be unified with QM. The model presented here suggests that the ansatz of Kaluza is sufficient to give an excellent model for particles, in particular in combination with the boundary condition spin, bypassing QM in 1st approximation [40]. The major deviation from conventional GR is dropping the constant of gravitation in the field equations, a minor thing from a mathematical point of view. The resulting objects of interest are waves only, which naturally fits basic concepts of QM. General features of quantum mechanics that emerge from such an ansatz include quantization of energy or the pivotal constant of quantum mechanics, Planck's constant, h, that may be derived from the electromagnetic constants and the expression of α in terms of gamma-functions.

QM may be seen as an effective theory where the spatial distribution of energy density of GR is replaced by a single parameter "mass" and the wave function represents actual wave-like states. Since QM is background dependent and curvature of space-time from the view of the model presented here is not negligible but the dominating effect as far as particles are concerned some concepts of QM might need reconsideration.

Comparing with the quantum field theory (QFT) of the standard model of particle physics (SM):

The results of the quaternion ansatz of chpt. 4 reproduce the set of elementary fermions of the SM. The number of 12 basic building blocks of matter can be traced back to the 3 possibilities to single out one of the orthogonal EBC-vectors and in a broad sense is a consequence of the 3 space dimensions in 4D space-time. While in the SM the properties of quarks, such as partial charges, are deduced from experimental particle data they can be *derived* in the quaternion ansatz. Leptons are an integral part of the particle classification scheme.

There are several features of the model that indicate a close relationship with electroweak theory. In addition to the obvious common root in EM there are: SU(2) symmetry, the energy of the Higgs boson /VEV as upper limit for particle energy and the electroweak coupling constants as central parameters. As for chirality the inherent chiral character of a circular polarized EM-wave is transferred via the orthogonal EBC-triple of the quaternion ansatz to particles.

39 Experimental values: PDG [7]: $\sin^2\theta_W = 0.231$, CODATA [10]: $\sin^2\theta_W = 0.222$).

40 QED terms are considered to be a necessary correction for the results of this model.

On the other hand, there seems to be no deeper connection with the concepts of quantum chromodynamics (QCD), such as color [41] or gluons. Properties such as confinement or the need for adhering to the Pauli principle in e.g. the Δ^{++} are obsolete for an object that is an electromagnetic wave. The development of the SM from constituent quarks towards QCD, based on valence and sea quarks plus gluons, was in part required by the limitations in explaining scattering experiments with 3 point-like objects only. The waves of this model are consistent with a point like structure function and still feature spatial extension.

Thus not all details of the SM are reproduced by the particle model presented here. However, the relevant benchmark is the agreement with experiments and as for the aspects examined up to now and described above this modified Kaluza model tends to exceed the capabilities of the SM considerably, last not least in regard of the number of free parameters needed: zero. Preliminary results for additional properties such as particle decay or scattering seem to be promising as well (see e.g. [A6]).

The origin in the formalism of GR is a particular strong point since it allows to use the same concepts and parameters from the particle to the cosmological scale. A fundamental model for elementary mass should be expected to yield some information about mass interaction. In reverse, since the latter exists in form of the concepts of GR it might be no surprise that these concepts are useful to describe mass itself. That the relationship involves a simple series expansion in turn raises expectations that the following term captures deviations from the simple laws of gravitation.

The 3rd order term is interpreted as a 4D-de Sitter space, a subspace of the flat 5D-Minkowski space-time of this modified Kaluza ansatz. A close connection of 4D-de Sitter space, cosmological constant and astrophysical phenomena described by MOND has been proposed by M.Milgrom since the 1980s and is reflected in the equations of this model.

Minor assumptions and coefficients take up a lot of space in this work, with the aim of achieving the best possible agreement with experiments and observations. However, already the most basic framework gives reasonable approximations. The BTFR, shown in fig. 2, is a typical example for the efficiency of this model:

Even the simplest relationship ($v_f^4 \approx ß^4 c_0^4\ M_G/M_e$) gives results in the correct order of magnitude, while one or two additional assumptions may lead to a precision in the range of a few percent. Moreover, since $ß^4 = (M_e/M_{Pl})^4 = (F_G/F_C)^2$ holds, not only electron mass is obviously the only input parameter but one can also easily identify the underlying series expansion and might infer that the combination of electrostatic potential times some exponential function could be useful to calculate particle energy as well.

Overall, with the modified Kaluza model it is possible to get results that cover about 5 decades in order of magnitude of energy density or length within an error of factor of 2 [42], see fig. 4.

Many details still need significant improvement. The simple ansatz used requires sufficient separation of length and interaction scales and is not expected to cover intermediate cases such as the transition between Newton and deep-MOND behaviour. Accuracy seems to be limited by the appropriateness of the incomplete gamma functions, e.g. choice of different functions, results compared to input without gamma functions and the general question how accurately the approximations that yield the simple exponential of Φ represent the actual state. A more specific version of the metric is needed that includes the magnetic potentials and accounts for spin to further clarify various aspects, e.g. whether the terms of the series expansion can be directly attributed to 1st and 2nd derivatives of the exponential function in the metric.

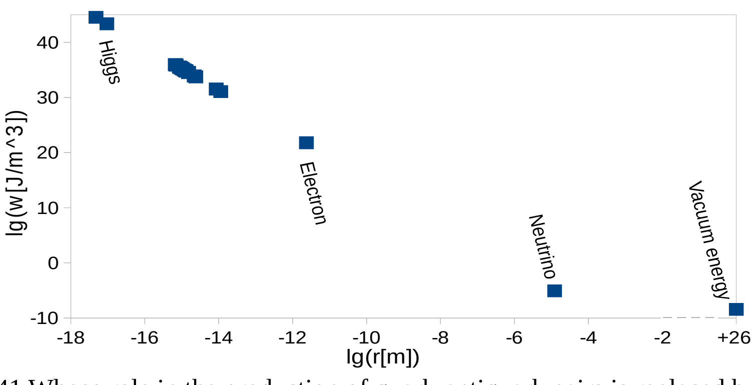

Fig. 4: Energy density, w, vs. length parameter, r (using Compton wavelength and $\Lambda^{-0.5}$), for states described by this model; agreement with experiments and

41 Whose role in the production of quark-antiquark pairs is replaced by chiral pairs, see chpt. 4.3.

42 Electron, Higgs-VEV/Boson energy and BTFR give calculated values within a factor of 2 of the experimental ones, as does values for vacuum energy/cosmological constant if back calculated from the BTFR.

observations within factor of 2 (ex. *v*);

Conclusion

A formalism based on a Kaluza ansatz with spin as boundary condition provides a simple, coherent, comprehensive and first of all quantitative description of phenomena in physics that covers a range from the particle to the cosmological scale, including:

- a set of 6 lepton-like and 6 quark-like objects with the associated charges, defined by 3D-space and spin 1/2,
- a convergent series of particle energies quantized as a function of the fine-structure constant, α, spanning the range from the energy of the electron to the Higgs VEV,
- magnetic moments of baryons,
- a single expression for the values of electroweak coupling constants,
- a series expansion for Kaluza's scalar Φ that allows to identify the first 3 terms as representing electromagnetic + gravitational + dS_4-background -contributions,
- values of Λ_0 and a_0 of MOND/BTFR in the correct order of magnitude.

The model works *ab initio* without free parameter and allows to remove some values from the set of fundamental constants:

electromagnetic constants, h, G, α, α_{weak}, energies of elementary particles => electromagnetic constants.

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Appendix

In the following the exponential part of Φ^2 is abbreviated as e^v.

[A1] Scalar potential Φ

Equ. (5) may in general be interpreted to refer to the highest order terms of exponential N in Φ'':

$$\Phi_N{}'' \sim \left(\frac{\rho^{3N-1}}{r^{3N+1}}\right)e^{v/2} \sim \boldsymbol{\Phi_N^3}\ e^{-v}\left(\boldsymbol{A_{el}}'\right)^2 \approx \left[\left(\frac{\boldsymbol{\rho}}{\boldsymbol{r}}\right)^{\boldsymbol{N-1}}\boldsymbol{e^{v/2}}\right]^3 e^{-v}\left(\frac{\boldsymbol{\rho}}{\boldsymbol{r^2}}\right)^2 = \left(\frac{\rho}{r}\right)^{3N-3}e^{v/2}\left(\frac{\rho}{r^2}\right)^2 \quad (60)$$

The solutions for the scalar Φ depend on the complete metric used. The easiest method to get a solution of order N is to use spherical coordinates of dimension N+1. Using e.g. the line element for a 4D metric of [4, equ. 6.76]

$$ds^2 = e^v dt^2 - e^\lambda dr^2 - e^\mu r^2\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right) \quad (61)$$

and $A_\alpha = (A_{el}, 0, 0, 0)$ gives as solution for equ.(4) (cf. [4, equ. 6.77], prime corresponds to derivatives with respect to r):

$$\Phi'' + \left(\frac{v' - \lambda' + 2\mu'}{2} + \frac{2}{r}\right)\Phi' - \frac{1}{2}\Phi^3 e^{-v}\left(A_{el}'\right)^2 = 0 \quad (62)$$

This can be solved with a function of type (5) for N = 2:

$$\Phi_2{}' = \left[-\left(\frac{\rho}{r^2}\right) + 2\left(\frac{\rho^3}{r^4}\right)\right]e^v \quad (63)$$

and

$$\Phi_2{}'' = \left[2\left(\frac{\rho}{r^3}\right) - 10\left(\frac{\rho^3}{r^5}\right) + 4\left(\frac{\rho^5}{r^7}\right)\right]e^v \quad (64)$$

The ρ^1 terms cancel in (62), the ρ^3 terms can be eliminated by appropriate choice of v', λ' and μ', a remaining factor in the ρ^5 term could be compensated by assuming a corresponding factor in A_{el}. For N = 3 hyperspherical coordinates with the line element

$$ds^2 = e^v dt^2 - e^\lambda dr^2 - e^\mu r^2\left(d\psi^2 + \sin^2\psi\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right)\right) \quad (65)$$

may be used. A more complex metric of the kind given in [A2] may be used as well to solve equation (62).

[A2.1] Metric / point charge

[A2.1.1] Metric with equation (10) as solution

Equ. (10)f would be the result of the following metric (which does not account for magnetic potential or spin!):

$$g_{\mu\mu} = \left(\frac{\rho_0}{r}\right)^2\exp\left(-\left(\frac{\rho}{r}\right)^3\right)A_{el}^2,\ -\left(\frac{\rho_0}{r}\right)^2\exp\left(\left(\frac{\rho}{r}\right)^3\right)A_{el}^2,\ -r^2A_{el}^2,\ -r^2\sin^2\vartheta\, A_{el}^2 =$$
$$g_{\mu\mu} = \left(\frac{\rho_0}{r}\right)^4\exp\left(-\left(\frac{\rho}{r}\right)^3\right),\ -\left(\frac{\rho_0}{r}\right)^4\exp\left(\left(\frac{\rho}{r}\right)^3\right),\ -\rho_0^2,\ -\rho_0^2\sin^2\vartheta \quad (66)$$

[A2.1.2] Metric with typical extra terms

The following gives an alternate metric in some detail to illustrate the significance and order of magnitude of the relevant terms:

$$g_{\alpha\alpha} = \left(\frac{\rho_0}{r}\right)^2\exp\left(-\left(\frac{\rho}{\mathbf{r}}\right)^3\right),\ -\left(\frac{\rho_0}{r}\right)^2\exp\left(\left(\frac{\rho}{\mathbf{r}}\right)^3\right),\ -r^2,\ -r^2\sin^2\vartheta \quad (67)$$

The variable r is marked bold if originating from the exponential term to facilitate a discussion of the implications of its restricted range of validity.

$\Gamma_{01}^0 = \Gamma_{10}^0 = -1/r^1 + 3/2\,\rho^3/\mathbf{r}^4$ $\qquad \Gamma_{00}^1 = -1/r^1 e^{-2v} + 3/2\,\rho^3/\mathbf{r}^4 e^{-2v}$

$\Gamma_{11}^1 = -1/r^1 - 3/2\,\rho^3/\mathbf{r}^4$

$\Gamma_{12}^2 = \Gamma_{21}^2 = \Gamma_{13}^3 = \Gamma_{31}^3 = +1/r^1$ $\qquad \Gamma_{22}^1 = -r^3/\rho_0^2\, e^{-v} = \Gamma_{33}^1/\sin^2\vartheta$

$\Gamma_{23}^3 = \Gamma_{32}^3 = \cot\vartheta$ $\qquad \Gamma_{33}^2 = -\sin\vartheta\cos\vartheta$

$R_{00} = e^{2v}\,[+1/r^2 + 6\,\rho^3/\mathbf{r}^5 - 9/2\,\rho^6/\mathbf{r}^8]$

$R_{11} = +3/r^2 - 6\,\rho^3/\mathbf{r}^5 + 9/2\rho^6/\mathbf{r}^8$

$R_{22} = -1 + e^{+v}\,[+r^2/\rho_0^2 + 3\rho^3 r^3/(\rho_0^2\mathbf{r}^4)]$

$R = +2/r^2 + e^v\,[(-4/\rho_0^2 - 6\rho^3 r/(\rho_0^2\mathbf{r}^4) + 12\rho^3 r^2/(\rho_0^2\mathbf{r}^5) - 9\,\rho^6 r^2/(\rho_0^2\mathbf{r}^8)]$

$G_{00} = e^{2v} [+1/r^2 + 6\rho^3/\mathbf{r}^5 - 9/2\rho^6/\mathbf{r}^8)] - e^v \rho_0^2/r^4 + e^{2v} [2/r^2 + 3\rho^3/(r\mathbf{r}^4) - 6\rho^3/\mathbf{r}^5 + 9/2\, \rho^6/(\rho_0^2\, \mathbf{r}^8)] =$
$- e^v \rho_0^2/r^4 + e^{2v} [3/r^2 + 3\rho^3/(r\mathbf{r}^4)]$

Volume integrals over any ρ^n/r^{n+2} term will yield energy results $\varepsilon_c \int e^v \rho^n/r^{n+2}\, d^3r \approx \varepsilon_c\, \rho \approx$ 1E-22 [J] compared to the term $\varepsilon_c \int e^v \rho_0^2/r^4\, d^3r \approx \varepsilon_c\, \rho_0^2\, \rho^{-1} \approx$ 1E-13 [J] (both with coefficients for the electron, σ_0ß) giving negligible contributions to particle energy within the parameter range discussed here. This leaves the first term as leading order: $G_{00} = - e^v \rho_0^2/r^4$.

[A2.2] General solution N = {1; 2; 3}

This article has a focus on a solution of (5) with N = 3. However, all solutions in a 5D space-time according to [A1], i.e. up to using hyperspherical coordinates, N = {1; 2; 3}, might be used for the ansatz of a metric such as

$$g_{00} = \sum_{N=1}^{3} \left(\frac{\rho_0}{r}\right)^{N-1} \exp\left(-\left(\frac{\rho}{r}\right)^N\right) \tag{68}$$

With the approximation $\sigma \approx 1$ and assuming an identical coefficient ß in each term this gives for g_{00}:

$$g_{00} = \exp\left(-ß\left(\frac{\rho_0}{\mathbf{r}}\right)\right) + \left(\frac{\rho_0}{r}\right)\exp\left(-ß\left(\frac{\rho_0}{\mathbf{r}}\right)^2\right) + \left(\frac{\rho_0}{r}\right)^2 \exp\left(-ß\left(\frac{\rho_0}{\mathbf{r}}\right)^3\right) \tag{69}$$

Each term might be expanded and split in EM and gravitational part as suggested in chpt. 3.
The 3rd term corresponds to the case discussed above, resulting in terms giving the square of the E-field in G_{00} and eventually particle energy as a kind of self energy. The second term is the linear version and might be used to construct solutions for potential terms. The first term might represent a general vacuum solution, i.e. without presence of any field ρ_0/r.

[A3] Model coefficients

[A3.1] Coefficient σ as component in ρ

The exponential term, $\exp(-\rho^3/r^3)$, together with the r^{-2} dependence of the field of a point charge define a maximum of particle energy near $r_{W(max)} \approx \rho$, rapidly approaching 0 for $r_{W(max)} > \rho$, effectively allowing to approximate particle energy without using a specific upper integration limit, r_n, see fig. 1. On the other hand the weaker r-dependence of angular momentum, ~1/r results in the calculated values being completely dominated by an integration limit. The limit of the Euler integral of a particle is given by ρ_n^3/r_n^3, a constant which will be denoted $8/\sigma$ in this work.
A general exponential function of radius featuring a limit radius, assumed to correspond to a damped oscillator-like solution and a discriminant term, may be given in 1st approximation as:

$$e^{v'} = \exp\left(-\left(\frac{ß\rho'^3}{2r^3} + \left[\left(\frac{ß\rho'^3}{2r^3}\right)^2 - 4\frac{ß\rho'^3}{2r^3}\right]^{0.5}\right)\right) \tag{70}$$

ß being some general coefficient. At the limit r_n of the real solution of (70)

$$\left(ß\rho'^3/r_n^3\right)^2 = 8\,ß\rho'^3/r_n^3 \quad => \quad ß = 8\left(\frac{r}{\rho'}\right)^3 = \sigma \tag{71}$$

holds, reproducing the definition of σ according to (15). Within the parameter range of this work for calculating particle energy the function $e^{v'} \approx \exp(-(ß\rho'^3/r^3))$ is a very good approximation of an equation of the kind of (70) and coefficient σ will have to be part of the exponential.
For numerical calculations using a term of type (70) and $r > \alpha r_n$, i.e. limits to calculate S_z or S, requires an additional factor to appear in the denominator of the linear term of the discriminant ($\approx \sigma_0$ in case of S_z).

[A3.2] Coefficient σ, coefficient 1.5x

The basic relation of α(n) and σ with the fine-structure constant α and coefficient $\Gamma_{-1/3}/3$ is due to the considerations of chpt. 2.4ff. To get a more detailed description in a range of 1% precision is difficult since there are several options conceivable and in this range of accuracy QED and other minor effects may be expected, which might be amplified due to the non-linear nature of the Γ-functions involved. A factor ≈ 3/2 appears in several terms such as $\sigma_0 \sim 1.5\alpha^{-1}$ of (6), the ratio of electron and muon energy =1.5088, $\Gamma_{-1/3}/\Gamma_{+1/3}$ =1.516, π/2 = 1.5707, $(4\pi)^{1/6}$ and the irregular electron coefficient in the power series that is part of ß as well. The following discusses some relevant aspects with a focus on identifying possible underlying relationships while minimizing assumptions about the term ≈ 3/2 in particular.
To get the value of e_c from (13) coefficient Γ(+1/3)/3 is required to appear as a term in $W(e_c)$ due to the Euler integral, thus a counter term would have to be part of ρ in (13):

$$W(e_c) = \frac{e_c^2}{4\pi\varepsilon_c}\int \exp\left(\frac{-\Gamma_{+1/3}}{3}\frac{e_c}{4\pi\varepsilon_c}\right)^3 r^{-2} dr = \frac{e_c^2}{4\pi\varepsilon_c}\frac{\Gamma_{+1/3}}{3}\left(\frac{\Gamma_{+1/3}}{3}\frac{e_c}{4\pi\varepsilon_c}\right)^{-1} = e_c \tag{72}$$

To deal with both $\Gamma_{-1/3}$ and $\Gamma_{+1/3}$ an additional term of 2π in the denominator of ρ and relation (78) might be useful, e.g. with coefficients of (23)ff:

$$\lambda_C \sim 3^{0.5} \int \exp - \left(\frac{\Gamma_{+1/3}}{2\pi 3} \frac{e_c}{4\pi\varepsilon_c} \right)^3 dr \sim \frac{\Gamma_{-1/3}\Gamma_{+1/3}}{2\pi 3^{0.5}} \frac{e_c}{4\pi\varepsilon_c} = \frac{e_c}{4\pi\varepsilon_c} \qquad (73)$$

Since according to 2.4.3 σ-terms should appear as $\sigma^{2/3}$ in a length expression and using the simplest version of σ, $\sigma_{lim} = (2\Gamma_{-1/3}/3)^3$ an additional term $((2\pi)^{-1}\Gamma_{+1/3}/\Gamma_{-1/3})^3$ (bold in (74)) in ρ would cancel redundant $\Gamma_{-1/3}/3$ terms in the corresponding length expression as well:

$$\lambda_C \sim 3^{0.5} \frac{\sigma_{\lim}^{1/3}}{2} \rho \approx \frac{3^{0.5}}{2} \left(\frac{2\Gamma_{-1/3}}{3} \right)^2 \frac{\boldsymbol{\Gamma_{+1/3}}}{\boldsymbol{2\pi\Gamma_{-1/3}}} \frac{e_c}{4\pi\varepsilon_c} = \frac{2}{3} \frac{e_c}{4\pi\varepsilon_c} \qquad (74)$$

A term $2/3(2\pi\Gamma_{-1/3}/\Gamma_{+1/3})$ consists of components related to angular momentum, appears in (81) and may thus be used in alternate expressions for σ_0.

[A3.3] S = √3/2 as limit of a particle state

One has to be careful to compare the relationship of S_Z and S in this model with the classic or quantum mechanical state. S_Z refers to a preferred orientation that might be relevant in a regular particle state, in particular in linked particle sub-states as discussed in [A3.5], due to necessary alignment of the internal fields. At the transition to a free point charge field these restrictions may be lost and S = √3/2 marks the absolute limit of a particle state. In addition:

- it is the value where particle coefficients cancel in the exponent of Φ, leaving the pure electric potential term,
- it is the approximate limit for a series expansion of the exponential of Φ to yield a gravitational term, chpt. 3.

The upper limit of the particle energy series, cf. [A7.3], the transition from a vector rotating on the surface of a cone to the vector itself, seems to be somewhat similar to the spin situation and shares the factor σ_{min}, however, the final state is considered to have S = 0.

The unique value for radius, r_l, defined by S = √3/2[ħ] in chpt. 2.4.4.1 allows to calculate ß directly with (29) by fitting for r_l, S = √3/2 or both. Depending on which parameters one sets the focus on and the preferred approach for σ_0, accuracy may vary in a range of ~ 1%, typical for this model [A3.4].

[A3.4] Choice of gamma-functions

The ratio of length values characterizing S=1/2 and S=√3/2 allows to assess possible differences due to the usage of Γ-functions:

- calculation of Γ(1.E-15) as approximation of Γ(0), using https://keisan.site/exec/system/1161228685: r_l/r_e = 54.136
(1.000077 relative to theoretical value √3/(8πα); own numerical calculation with Γ(0): 1.00015);
- ratio $\sqrt{3}e_c/(4\pi\varepsilon_c)/\lambda_{C,e}$: r_l/r_e = 52.960;
- numerical calculation of Γ(-1/3, x): result depends on error budget for W_e, S=√3/2[ħ]; r_l/r_e = 52.960
(Error W_e: 1.037, S=√3/2[ħ]: 5E-6);

[A3.5] Relationship with Lorentz boost / (Wigner-) rotation/spin

Interpreting the difference in wavelength of different states as a length contraction due to a Lorentz boost and calculating the necessary velocity according to $l = l_o(1-v^2/c_0^2)^{0.5}$, the ratio of 2 consecutive steps will converge to $v_n/v_{n+1} = 3^{0.5}$ for large n (i.e. small v_n). This is the ratio of the sum of 3 orthogonal vectors of equal length to a single vector, a simple vector addition that corresponds to a Wigner rotation in 3D for the non-relativistic limit [17]. By adding again 3 orthogonal vectors of the resulting vector sum (i.e. of length $3^{0.5}$ of the original vector) one may construct an infinite series of connected states.

The ratio of the vectors, $3^{0.5}$, is the same as that between total spin S = √3/2 and its z-component S_z =1/2, indicating that angular momentum and in particular alignment of magnetic moment / spin of sub-units of particle states may play a role. A connection with the [ħ]/2-√3[ħ]/2-relation in chpt. 2.4 might be possible.

The relation according to a Wigner rotation will be less simple for small n. A relationship such as given in (32) might describe a cascade of interrelated particle states that smoothly transforms into what conventionally would be considered the “field” of a particle.

[A4] Coupling constants in N dimensions

[A4.1] Fine-structure constant, α

Using equ. (23) for the incomplete Γ-function and multiplying r_x in the integration limit $(\rho_n/r_x)^3$ by √3 to obtain a term for Compton wavelength, $\lambda_{C,n}$, (cf. (26)), gives in good approximation:

$$\lambda_{C,n} \approx 3^{1.5}\, \sigma_0^{1/3}/2\, \rho_n/3 \qquad (75)$$

With (75) energy of a photon may be expressed as:

$$W_{Phot,n} = hc_0 \,/ \int^{\lambda_{C,n}} e^{-\left(\frac{\rho}{r}\right)^3} dr = \frac{2hc_0}{3^{0.5}\rho_n \sigma_0^{1/3}} \qquad (76)$$

Equating (14) with (76) gives:

$$W_{pc,n} = W_{Phot,n} = 2b_0\,\Gamma_{+1/3}\,\rho_n^{-1}/3 \;=\; \frac{2hc_0}{3^{0.5}\rho_n\sigma_0^{1/3}} \tag{77}$$

Solving equ. (77) for α will involve a term of two Γ-functions with an argument of same value and opposite sign for which the relation $\Gamma(+x)\Gamma(-x) = \pi/(x\sin(\pi x))$ holds [9], giving for the product $\Gamma_{+1/3}\Gamma_{-1/3}$:

$$\Gamma_{+1/3}\Gamma_{-1/3} = 3^{0.5}\,2\pi \tag{78}$$

Using equation (77) with (78) and (19) will give

$$\alpha^{-1} = \frac{hc_0}{2\pi b_0} = \left(\frac{\Gamma_{+1/3}}{3^{0.5}2\pi}\right)\left(\frac{2\Gamma_{-1/3}}{3}\frac{1.5}{\alpha}\right) = \left(\frac{1}{\alpha}\right) \tag{79}$$

This would justify the use of α in (19), however, the factor of 1.524 should be rather exactly 3/2. The resulting σ would be σ* = 1.725E+8 ≈ $0.95\sigma_0$. The precise result of 3/2 for factor ≈ 1.5 in (19)ff is actually misleading, since due to the integration limits in (12) the values for the gamma functions have to be rather those of the incomplete gamma functions, i.e. (79) can only hold approximately, cf. fig. 5.
An alternate, geometric interpretation gives a value closer to σ_0 of 2.4.2:

$$\sigma_0^* = 8\left(\frac{4\pi}{3}\;\Gamma_{-1/3}{}^3\right)^3 \quad = 1.7715\text{E+8} \tag{80}$$

that allows to obtain a volume-like term in the energy expression, a term of the kind of (20) might represent a 1D-term. The expression equivalent to (79) would be:

$$\alpha^{-1} = \frac{hc_0}{2\pi b_0} \approx \left(\frac{\Gamma_{+1/3}}{3^{0.5}2\pi}\right) 2\left(\frac{4\pi}{3}\Gamma_{-1/3}{}^3\right) \approx \frac{2}{3}\frac{\Gamma_{-1/3}}{\Gamma_{+1/3}}4\pi\,\Gamma_{+1/3}\Gamma_{-1/3} \approx \mathbf{4\pi\,\Gamma_{+1/3}\Gamma_{-1/3}} \tag{81}$$

Equations (79) and (81) do not necessarily contradict each other. In chapter 2.4.3 and [A3.2] reasons are given for the same gamma-functions to appear more than once in the relevant equations. Inserting the result of (81) in (79) and considering that at least one of the Γ(+1/3) Γ(-1/3) pairs has to refer to the integrals of (56)f, i.e. they have to be given as Γ(+1/3,x) Γ(-1/3,x) with values including the deviations at the integration limit of σ_0, see fig. 5, gives:

$$\alpha^{-1} = \frac{\Gamma(+1/3,x)}{3^{0.5}2\pi}\;\frac{2\Gamma(-1/3,x)}{3}\frac{3}{2}\;4\pi\,\Gamma_{+1/3}\Gamma_{-1/3} \tag{82}$$

As can be calculated or taken from fig. 5 the ratios of the gamma functions are approximately Γ(+1/3,x)/Γ(+1/3)= 0.9977; Γ (-1/3,x)/Γ (-1/3)=1.0154, $(4\pi\,\Gamma_{+1/3}\Gamma_{-1/3})/\alpha^{-1}$=0.9980. The product gives 1.011 which is 2/3 Γ(-1/3)/ Γ (+1/3) = 2/3 1.5164 of (81), i.e. ≈1.5 in (79) may be interpreted as the term Γ(-1/3)/ Γ (+1/3) of (81).

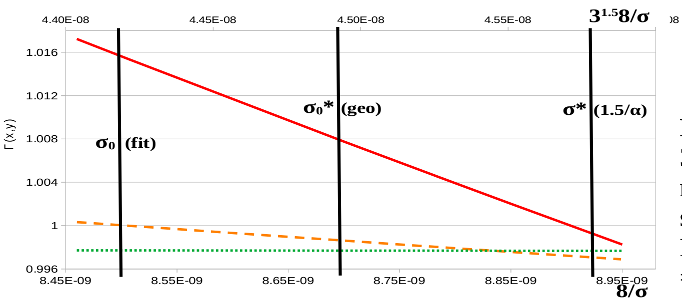


Fig. 5: Γ(+1/3, 8/σ) - green dots, lower x-scale / Γ(0, $3^{1.5}$8/σ) - orange dashes, upper x-scale / Γ(-1/3, 8/σ) - red continuous line, lower x-scale; black lines mark selected σ-values as discussed in the text; reference for Γ(0) is $\alpha^{-1}/(8\pi)$, for Γ(+1/3,x) => Γ(+1/3) for Γ (-1/3,x) => $36\pi^2\Gamma(-1/3)$ [43];

[A4.2] Coupling constants - geometry, integrals

The 3D case of the coupling constant is easy to interpret, for the 4D-case some assumptions have to be made concerning the integration limit. The following gives an alternative aproach for [A4.1] ($e^{v(N)} = \exp(-(\rho/r)^N)$).

3D case:

The exact value of the product of the integrals (41)f, depends on the integration limit relevant for the second integral, i.e. the lower integration limit of the Euler integrals, which may be expressed as 3D volume with $\Gamma_{-1/3}$ as radius (80):

$$\rho_n^3/\lambda_{C,n}^3 = 8/(3^{1.5}\sigma_0) = \left(3^{0.5}\frac{4\pi}{3}\;\Gamma_{-1/3}{}^3\right)^{-3} \tag{83}$$

The additional factor $3^{0.5}$ may be interpreted as the ratio between r_n and $\lambda_{C,n}$ as required in the expression for photon *energy* and given by(26), √3 is due to (78). This gives $\Gamma(-1/3, 1/\sigma_0) \approx 36\pi^2\Gamma_{-1/3}$ and

$$2\int_0^r e^{v(3)}r^{-2}dr\int_0^r e^{v(3)}dr \approx 2\left[\frac{\Gamma_{1/3}}{3}\right]\left[2\pi\,2\pi\,9\,\frac{\Gamma_{-1/3}}{3}\right] = 4\pi\,\Gamma_{1/3}\Gamma_{-1/3}\;\mathbf{2\pi} = \mathbf{2\pi}\;\alpha^{-1} \;\; ^{44} \tag{84}$$

43 Term $36\pi^2$ from $2b_0\Gamma_{+1/3}/3 = 2\pi h/2\pi\; c_0\,3/(36\pi^2\Gamma_{-1/3})$;
44 Factor 2 from adding electric and magnetic contributions to energy;

The result of (84) yields a dimensionless constant α' = hc_0 4π ε/e^2 and it may be seen as a matter of choice to include 2π in the dimensionless coupling constant. Factor 9 cancels the corresponding factors from the Euler integrals. The remaining factor of 4π is needed to yield the correct value of α.

A general N-dimensional version of (83) may be given as:

$$8/\sigma_N = \left(3^{0.5\,\delta} V_N \,\left(\Gamma(-1/\mathrm{N})\right)^N\right)^{-N/(N-2)} \quad (85)$$

V_N is the coefficient for volume in N-D, coefficient $3^{0.5}$ will be omitted in 4D where coordinate r is assumed to be directly related to energy via $r_n \sim 1/W_n$ and r_n might be directly identified with $\lambda_{C,n}$; the equivalent of (78) would not involve √3 but give 2π; subscript in σ_N corresponds to dimension in the following.

4D case:

Using $e^{v(4)}$ according to the definition (5) and (85) for 4D:

$$\rho_n^4/r_n^4 = 8/\sigma_4 = \left(\frac{\pi^2}{2}\,\left(\Gamma_{-1/4}\right)^4\right)^{-2} = 1.232\text{E-}7 \quad (86)$$

as integration limit, with (12) the non-point-charge integral in 4D will be given by:

$$\int_0^r e^{v(4)} r\, dr \sim \Gamma\left(-1/2, 8/\sigma_4\right) = \int_{8/\sigma_4}^{\infty} t^{-1.5} e^{-t}\, dt = 5687 \approx 16\,\pi^4\,\Gamma_{-1/2} \quad (87)$$

The 4D equivalent of (84) will be:

$$2\int_0^r e^{v(4)} r^{-3} dr \int_0^r e^{v(4)} r\, dr \approx 2\left[\frac{\Gamma_{1/2}}{4}\right]\left[16\pi^4 \frac{\Gamma_{-1/2}}{4}\right] = \frac{\pi^2}{2}\Gamma_{1/2}\Gamma_{-1/2}\ \mathbf{4\,\pi^2} = \pi^3\,\mathbf{4\,\pi^2} = \alpha_{weak}^{-1}\mathbf{4\,\pi^2} \quad (88)$$

The interpretation is the same as in the 3D-case:

A $4\pi^2$ term originating from the second integral of equation (88) is required for turning h^2 into $\hbar^2$ since the integral refers to ρ_n^2 and thus to the square of energy and h, ħ. Factor 16 cancels the corresponding factors from the Euler integrals. The remaining factor of $\pi^2/2$ is needed to yield the correct value of α_{weak}.

2D case:

the 2D case is not as straightforward as the 4D case. The integral over the 1D point charge

$$\int_0^r e^{v(2)} r^{-1} dr = \Gamma\left(0, \rho_n^2/r_2^2\right)/2 \quad (89)$$

features Γ(0, x), with Γ(0, x) -> ∞ for x -> 0 and m = N-2 = 0 in the equations above. Setting nevertheless m=1 in the 2D equivalent of the integration limit

$$\rho_n^2/\lambda_{C,n}^2 = 8/(\sigma_2) = \left(3^{0.5}\pi\ \ \Gamma_{-1/2}{}^2\right)^{-2} \approx 1/4676 \quad (90)$$

and calculating $\Gamma(0, \rho_2^2/r_2^2)$ numerically gives $\int e^{v(2)} r^{-1}\, dr \approx \Gamma(0, \rho_2^2/r_2^2)/2 = 7.872/2$. In the 2D case the complementary integral would be identical to the point charge integral, giving $2(\int e^{v(2)} r^{-1}\, dr)^2 \approx 4\pi^3/4 = \pi^3$, i.e. the same value as 4D, maybe giving an alternate candidate for α_{weak} .

That α and α_{weak} are somehow related may be expected in this model. Weak processes such as ß-decay, involving switching of charge, would require an actual inversion of the E-field vector, i.e. something that might be described in terms of a rotation in 5D.

[A5] Quaternion-based quark-like model

[A5.1] Quaternion UDS-components

In the following the model described in chpt. 4 will be explained in some more detail. A standard algorithm for rotation with quaternions will be used.

Three orthonormal vectors E, B, C described as imaginary part of a quaternion with real parts set to 0, will be subject to alternate, incremental rotations around the axes E, B and C. For each E, B and C the following variables will be defined:

- de, db, dc: incremental step for rotation angle,
- de_sum, db_sum, dc_sum: total rotation angle,
- ex, ey, ez, bx, by, bz, cx, cy, cz: Cartesian components of the respective vectors,
- eex, eey, eez, bbx, bby, bbz, ccx, ccy, ccz: Cartesian components of the respective vectors to be buffered until rotation around the axes E, B and C is complete,
- sih, qw, qx, qy, qz: internal variables for quaternion-rotation calculation.

The following part of the algorithm gives the rotation of B around the E axis for an incremental step de:

```
de_sum = de_sum + de;   sih = sin(de / 2);   qw = cos(de / 2);    qx = ex * sih    qy = ey * sih;    qz = ez * sih;
bx = bbx;    by = bby;    bz = bbz;
bxx = bx * (qx * qx + qw * qw - qy * qy - qz * qz)  + by * (2 * qx * qy - 2 * qw * qz) + bz * (2 * qx * qz + 2 * qw * qy);
```

byy = bx * (2 * qw * qz + 2 * qx * qy)   + by * (qw * qw - qx * qx + qy * qy - qz * qz) + bz * (-2 * qw * qx + 2 * qy * qz);
bzz = bx * (-2 * qw * qy + 2 * qx * qz) + by * (2 * qw * qx + 2 * qy * qz) + bz * (qw * qw - qx * qx - qy * qy + qz * qz);
bx = bxx; by = byy; bz = bzz;

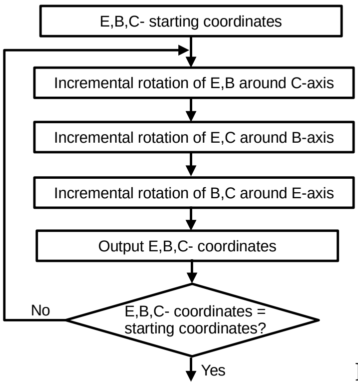


Fig. 6: Flowchart quaternion calculation

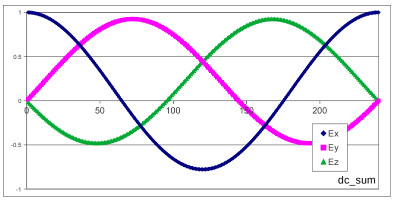
This has to be followed by rotation of C around the E axis and equivalent routines for the rotation of E, B around the C axis and the rotation of E, C around the B axis. After each incremental step for de, db, dc the Cartesian components of the E, B, C vectors may be stored in a list.

The vectors are thought to indicate spatial orientation only, *polarity (sign) of E and B has to be considered in the analysis of the results*. Orientation of angular momentum remains a free parameter.
In the following only solutions where one of the incremental angles of rotation has half the value of the other two will be considered. This may serve as a primitive model for spin S = 1/2.
There are 6 possible solutions for de, db and dv, respectively, to be called U, D, S, C, B, T:

	de = 0.5 db = 0.5 dc				de = 0.5 db = 0.5 dc				de = 0.5 db = 0.5 dc			
	E-comp	E-avg	B-comp	B-avg	E-comp	E-avg	B-comp	B-avg	E-comp	E-avg	B-comp	B-avg
Spherical cone	2/9, 2/9, 1/9	1/3	4/9, 4/9, 2/9	2/3	4/9, 4/9, 2/9	2/3	2/9, 2/9, 1/9	1/3	4/9, 4/9, 2/9	2/3	4/9, 4/9, 2/9	2/3
	U				D				S			
Toroidal wedge	4/9, 4/9, 2/9	2/3	2/9, 2/9, 1/9	1/3	2/9, 2/9, 1/9	1/3	4/9, 4/9, 2/9	2/3	2/9, 2/9, 1/9	1/3	2/9, 2/9, 1/9	1/3
	C				B				T			

Tab. 4: Average of x,y,z-components (E,B-comp) and total average (E,B-avg) of E-and B-field for complete rotation;

The average of the x, y, z-components of the fields are multiples of 1/9th of the original vector length, the average total sum of E- and B-fields is 1/3 or 2/3. Surface area / fractional charge of 1/3 and 2/3 correspond to an average of the E-field of 2/3 and 1/3.
The diagram for the E,B, C-components as function of the angle dc_sum is given in fig. 7a for a U-entity.
From a coordinate transformation to a representation with one Cartesian coordinate as axis of rotation (in fig. 7b transformation of z-axis +26,6°, x-axis -41,8°, to give y-axis as axis of rotation) one can infer that the E-vector circumvents a spherical cap of area 2πr (2/3)r. Mirroring at the center of rotation gives a value of 2/3 of the surface of a sphere, which according to Gauss' law may represent 2/3 of a full point charge. The analogue procedure yields a value of 1/3 of a point charge for D and S-rotations.



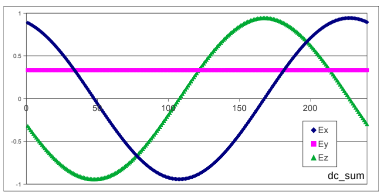


Fig. 7: a) E-components for Cartesian starting values   b) E-components after coordinate transformation

[A5.2] Magnetic moments of baryons from U, D, S-components [45]

To calculate magnetic moments of uds-baryons three components of U,D,S will be combined that represent orthonormal starting conditions for E, B. Spin/angular moment of the 3 components has to add up to S_Z = 1/2. Within this model this is not an assumption but may be calculated in principle in detail. In the following it will be sufficient to have two components sharing the same orientation of the axis of rotation, i.e. both can be transformed according to fig. 7 above with the same set of rotation angles, or - in a trivial case – to have 2 identical components. Together with the freedom in choosing direction of rotation, allowing for cancelling or adding up spin as needed, this will be sufficient to model S_Z = 1/2 baryons. Table 5 gives an example for UUD and DDU.

45 Note: to allow for comparison with tabulated values of M in units of [Am^2] the calculations in this chapter and in chpt. 4.2 use e [C] not e_c [J], conversion factor: [m^2C/s] /[$m^2 J/s$] = e/e_c = 1/19.4 [C/J].

In D_inv and U_inv the sign of E- and B-components is inverted. The D and U for calculation of the effective B-field include the appropriate sign from their charge while U_inv, D_inv components represent the actual geometric orientation of the E, B-vector only, which is needed for calculation of the angular momentum S from the square of the electromagnetic fields. In table 5 ”Rot_X_axis” and ”Rot_Z_axis” give the angle of rotation needed to transform to a representation with y-coordinate as axis of rotation for the B-field. For U_1 and D_inv of the proton as well as for D_2 and U_inv of the neutron the angles of transformation are identical, so is their transformed y-axis, i.e. they posses an identical axis of spin (average of B) while still maintaining their orthonormal relationship (B(t)). Since orientation of rotation is a free parameter opposite spin may cancel both contributions, leaving the 3rd component’s spin of $S_Z = 1/2$ as total spin of the nucleon.
The U and D components of proton / neutron have equal sign and relative value of the components of the E- and B-fields (given in tab. 5 only for the Bx, By, Bz-components (bold) relevant for calculating a geometry-based average value of B, B_Avg). The results for U and D are exceptional in regard to this exchangeability of U and D-components. For other particle pairs this is difficult to assess due to identical B-field components of U and S and the different internal symmetry of S-components compared to U and D [46].
In the case of the solutions examined, compliance with condition $S_Z = 1/2$ for the lambda-particle (UDS) can be maintained by using a spin-cancelling UD-solution in combination with an S-component, for UUS, DDS, USS-combinations trivial solutions with two identical components exist, in the case of DSS, Xi^-, one might resort to the method used for the nucleons to find a $S_Z = 1/2$ solution. Results for the best fitting appropriate UDS-combinations are shown in tab. 6.

	UUD	Proton		DDU	Neutron	
	U_1			D_1		
Start value	-Ez	-Bx	Cy	-Ex	-Bz	Cy
Bx, By, Bz	**-0.444444**	**0.444444**	**-0.222222**	**-0.222222**	**0.222222**	**-0.111111**
	E	B		E	B	
Rot_Z_axis	-45	135		-45	135	
Rot_X_axis	19.47	19.47		19.47	19.47	
	U_2			D_2		
Start value	-Ex	By	-Cz	Ey	-Bx	-Cz
Bx, By, Bz	**-0.222222**	**0.444444**	**-0.444444**	**-0.111111**	**0.222222**	**-0.222222**
	E	B		E	B	
Rot_Z_axis	-26.57	116.56		-26.57	116.56	
Rot_X_axis	41.82	41.81		41.82	41.81	
E, B inverted	D_inv			U_inv		
Start value	-Ey	-Bz	Cx	-Ez	-By	Cx
	E	B		E	B	
Rot_Z_axis	-45	135		-26.57	116.56	
Rot_X_axis	19.47	19.47		41.82	41.82	
	D			U		
Start value	Ey	Bz	Cx	Ez	By	Cx
Bx, By, Bz	**0.222222**	**0.222222**	**0.111111**	**0.444444**	**0.444444**	**0.222222**
Bx, By, Bz Avg(UUD)	-0.148148	0.37037	-0.185185	0.037037	0.296296	-0.037037
B_Avg			**0.439790**			**0.300890**

Table 5: Example for appropriate combinations of U- and D-components for proton and neutron;

	USD	Lambda		UUS	Sigma +		DDS	Sigma -		USS	Xi 0		DSS	Xi -	
	U			U			D			S			S		
Bx, By, Bz	-0.444	0.444	-0.222	-0.222	0.4444	-0.444	-0.111	-0.222	0.222	-0.222	-0.444	-0.444	0.444	-0.222	0.444
	S			U			D			S			S		
Bx, By, Bz	0.444	-0.444	0.222	-0.222	0.4444	-0.444	-0.111	-0.222	0.222	-0.222	-0.444	-0.444	-0.444	0.444	-0.222
	D			S			S			U			D		
Bx, By, Bz	0.222	0.222	0.111	0.4444	0.4444	0.222	0.444	0.444	0.222	0.444	0.444	0.222	0.222	-0.222	0.111
Bx, By, Bz Avg(UUD)	0.074	0.074	0.037	0.000	0.444	-0.222	0.074	0.000	0.222	0.000	-0.148	-0.222	0.074	0.000	0.111
B_Avg			**0.111**			**0.497**			**0.234**			**0.267**			**0.134**

Table 6: Combinations of UDS-components for calculating magnetic moments of baryons.

To calculate magnetic moments, above factors of B_avg, derived from the purely geometric quaternion model, have to be multiplied by a factor considering the absolute strength of fields. Using a simple model for a current loop, M = I*S (current * area), gives equ. (91) for magnetic moments of baryons with $S_Z = 1/2$.

$$\boldsymbol{M_n} \approx e c_0 \lambda_C / 2 \; * \; \text{B\_avg} \quad (= 2\pi \mu_B * \text{B\_avg}) \tag{91}$$

see tab. 7. Factor 2π in the Bohr magneton, μ_B, applicable for the electron and muon, is considered to represent a degree of rotational freedom of simple particles that more complex structures composed of several U, D, S-components might not exhibit, requiring 2π to be cancelled.
Results of table 7 are obtained from a large set of solutions, thus the statistical significance is low and a more

46 U and D are symmetric in their mutual E and B-fields while in S-components E- and B-fields are symmetric to each other.

comprehensive study of the appropriate combination of spin-components is needed. Control samples have been made to check that a) in rare cases where U and D solutions do not match a UUD/DDU pair the condition for S = 1/2 is not met; b) all U and D- components shown in the tables in combination with an S are components appearing in UUD/DDU-pairs as well.

		λ_C	e c_0 *λ_C /2	B_Avg	|M|Calc = e$c_0\lambda_C$ B_{avg}/2	|M|Exp[Am^2]	|M|Calc/ |M|Exp	|M|Calc/|M|Exp Const. quark
p^{+-}	UUD	1.32E-15	3.17E-26	0.440	1.39E-26	1.41E-26	0.988	-
n	DDU	1.32E-15	3.17E-26	0.301	9.55E-27	9.66E-27	0.988	0.973*
Λ^0	UDS	1.10E-15	2.64E-26	0.111	2.94E-27	3.10E-27	0.949	-
Σ^+	UUS	1.04E-15	2.50E-26	0.497	1.24E-26	1.24E-26	1.002	1.090
Σ^-	DDS	1.04E-15	2.50E-26	0.234	5.83E-27	5.86E-27	0.994	0.897
Ξ^0	USS	9.43E-16	2.26E-26	0.267	6.05E-27	6.31E-27	0.958	1.152
Ξ^-	DSS	9.38E-16	2.25E-26	0.134	3.01E-27	3.06E-27	0.983	0.784

Table 7: Magnetic moments for UDS-Baryons; col.3: Compton wavelength [7]; col.4: magnetic moment for current loop; col.5: average B-component from quaternion calc.; col.6: calculated magnetic moments; col.7: values from experiment [7]; col.8: ratio calculated / experiment value; col.9: ratio (calculated constituent quark model, [7]) / experiment [7]), *calc. via Clebsch-Gordan coefficients relative to p; Σ, Ξ via fit based on p, n, Λ^0.

[A5.3] Ratio of magnetic moments

The calculation of the ratio of magnetic moments is particularly simple and may be based on geometry only. In the quaternion model both E- and B-fields are oriented to the center (magnetic monopole character on particle level) and will feature average fields of 1/3 and 2/3 for quark-like objects. The B-field for u- and d-entities will have Cartesian components of ± 2/9, ± 2/9, ± 1/9 (d) and ± 4/9, ± 4/9, ± 2/9 (u). Unique solutions [47] for B-field components of nucleons will be e.g. ($B_{avg}=((\sum x_i)^2+(\sum y_i)^2+(\sum z_i)^2)^{0.5}/3$):

proton - uud -4/9, -4/9, -2/9 / -2/9, -4/9, -4/9 / +2/9, -2/9, +1/9 $B_{avg} = 141^{0.5}/27 \approx 0.440$

neutron - ddu -2/9, -2/9, -1/9 / -1/9, -2/9, -2/9 / +4/9, -4/9, +2/9 $B_{avg} = 66^{0.5}/27 \approx 0.301$

The ratio of both values is $(141/66)^{0.5}$ = 1.461631, which compared to the ratio from experiments [7] gives 1.461631 / 1.459898 = 1.001187.

These solutions are distinguished by one U and one D-component being collinear [48], indicating a particular stable configuration involving oppositely charged components (see [A6]).

Table 8 compares some ratios of baryon isospin pairs for calculations with the average of the B-field as calculated in [A5.2] with B_avg, i.e. geometry only, and with the experimental value of the Compton wavelength/particle energy.

	U,D,S-components	|M|Calc (λ_C exp)	B_avg
M(p/n)_Calc/M(p/n)_Exp	UUD/DDU	0.999809	1.001187
M(Σ^+/Σ^-)_Calc/M(Σ^+/Σ^-)_Exp	UUS/DDS	1.007813	1.001111
M(Ξ^0/Ξ^-)_Calc/M(Ξ^0/Ξ^-)_Exp	USS/DSS	0.974652	0.969601

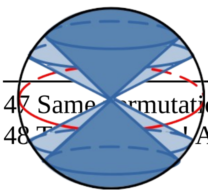
Table 8: Ratio of particle magnetic moments of baryon isospin pairs compared for calculated and experimental values [7] (col.4: geometry only, B_avg; col.3 inc. experimental particle energy);

[A6] Nucleons – stability, bonding in nuclei, scattering

Apart from the quantitative results for partial charges and magnetic moments some qualitative trends for nucleon properties may be inferred from the quaternion-based model.

The spin-cancelling of a UD-unit involves 2 collinear components with opposite charges occupying approximately the same spatial area (fig. 8), which is energetically favorable. This suggests among other things:

1) Comparatively lower energy for particles with UD-component;
2) High stability / life time of the nucleons;
3) A possible contribution to bonding in nuclei via UD-U—D-UD, a direct U-D-bond even without meson intermediate;
4) If such an inter-nucleon UD-bond plays a role in bonding in nuclei this would suggest a significant change in UD-structure between isolated and bound nucleons, which might play a role in the “EMC-effect” [18];
5) In DIS-experiments the ratio of the structure functions of neutron and proton, $F_2^n(x)/F_2^p(x)$ approaches 1 for x -> 0 (x = Bjorken-scale) which would be in agreement with a resolution of identical E and B fields of the EBC-triple of the nucleons rather than the averages of their U or D-units. For x -> 1 this model predicts the ratio $F_2^n(x)/F_2^p(x)$ to approach
$(z(UD)^2 + Z(D)^2)/(z(UD)^2 + Z(U)^2) = ((+1/3)^2 + (-1/3)^2)/((+1/3)^2 + (+2/3)^2) = 2/5$
which is in good agreement with high precision scattering experiments that yield values in the range 0.4 – 0.5 [19].



47 Same permutations and signs for u- and d-components; unique except for arbitrary orientation in space;
48 T[illegible] All E,B-components involved are orthogonal at any given point in time.

Fig. 8: Schematic illustration of a UD-unit

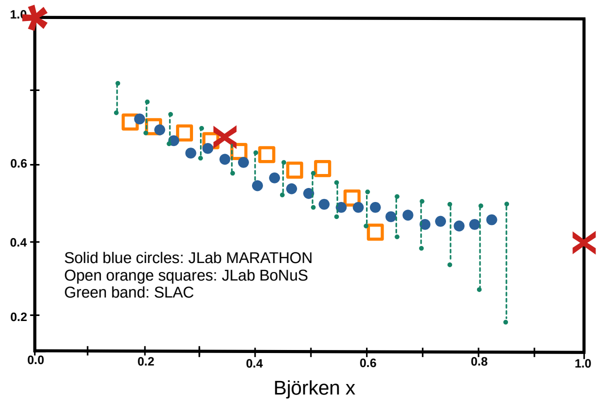


Fig. 9: Ratio of nucleon structure functions (Data from [19]); red crosses: values according to partial charges only, at x≈1/3 according to U+U+D and D+D+U units (2/3), at x=1 according to UD-units (2/5); Star at x=0 corresponds to an identical field distribution of E and B-fields in the nucleons, where the time average of E and B fields resulting in structures such as given in fig. 3, fig. 8 will be replaced by E(t) and B(t), both of identical strength for U and D.

[A7] Additional particle states

Assignment of more particle states will not be obvious. The following gives some possible approaches.

[A7.1] Partial products

One more partial product might be inferred from considering the next spherical harmonic, y_2^0, with a factor of $(2l+1)^{1/3} = 5^{1/3}$ as energy ratio relative to η, giving the start of an additional partial product series at $5^{1/3}$ W(η) = 937MeV i.e. close to energy values of the first particles available as starting point, η', Φ^0. However, in general it is not expected that partial products can explain all values of particle energies.

[A7.2] Linear combinations

Though the model reproduces basic properties of the quarks the fundamental differences might offer some alternate interpretations based on extended, non-point-like objects.

Linear combination states of the kaons, the first particle family that does not fit to the partial product series scheme, (34), and the η-particle might be an example for such an interpretation:

The kaons are designated to the linear combination of $(d\bar{s}$ +/- $\bar{d}s)/\sqrt{2}$ in the SM. They might be considered to be a linear combination of 2 extended y_1^0 states (double cones of $s|\bar{d}$, $\bar{s}|d$, etc., composition with 1 angular node) similar to the linear combination of 2 atomic p-orbitals, assumed to exhibit 2 angular nodes. A linear combination which would yield the basic symmetry properties of the 2 neutral kaons would be a planar structure such as:

$\bar{s}$ d
K_S^o d   s K_L^o s $\bar{s}$
$\bar{d}$ $\bar{d}$

providing two neutral kaons of different structure and parity (considering either flavour or chirality), implying a decay with different parity and lifetime.

A linear combination of 3 such states i.e. 3 orthogonal y_1^0 states would imply an approximate spherical symmetric object which might be attributable to the η-particle $((u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6})$.

[A7.3] Higgs boson

The considerations of chpt. 2.5.3 give the Higgs VEV as upper limit, the Higgs boson is very close to half its energy value. The "rotating E-vector" of chpt. 4 may be interpreted to cover the whole angular range in the case of y_0^0 of e.g. e or μ, while a y_1^0 object might be interpreted as forming a double cone. Increasing the number of angular nodes would close the angle of the cone leaving in the limit l -> ∞, a state of minimal angular extension representing the original E-vector. This may imply that essentially no space is left for rotation (i.e. Spin = 0) and a vanishing contribution of the magnetic field to total particle energy according to (14), resulting in a factor 1/2 and giving the Higgs *boson* as alternate upper limit of energy.

With the experimental value of W_e this gives $W_{Higgs\text{-}Boson} \approx W_e\, 9/8\, \alpha^{-2.5}$ = 2.025E-8 [J] (= 1.008 of experimental value); with equ. (33) $W_{Higgs\text{-}Boson} \approx$ 2.053E-8 [J] (= 1.022 of experimental value).

[A8.1] Cosmological constant Λ from minor terms in the metric

The terms in chpt. 3.3 are related to possible terms originating from a corresponding metric of the generic type as described in [A2] that will in general produce minor terms that might be considered as a natural candidate for e.g. the cosmological constant term, $g_{\alpha\beta}\Lambda_0$ as well. In [A2.1.2] an example is given to illustrate the emergence of typical extra terms. These will feature the same coefficients as the series expansion and thus might give equivalent terms for Λ_0 directly from the EFE. In particular terms such as $\rho_n^3/\mathbf{r}^5$ or $\rho_n^6/\mathbf{r}^8$ with all r originating from derivatives of the exponential only [49] will yield approximate values in the order of magnitude of critical, vacuum density, ρ_c, ρ_{vac} if setting $r_l = e_c/(4\pi\varepsilon_c)$ as upper bound of r.

$$\frac{\Phi''}{\Phi} \approx \frac{\rho^3}{\mathbf{r}_l^5} \approx \frac{ß}{(e_c/(4\pi\varepsilon_c))^5}\left(\frac{e_c}{4\pi\varepsilon_c}\right)^3 = 0.089\ [\mathrm{m}^{-2}] \qquad (92)$$

Multiplied by ε_c this gives an energy density of 2.97E-10 [J/m^3], multiplied by $8\pi G/c^4$ this gives for Λ: $\Lambda_{0,calc}$=6.17E-53[m^{-2}] $\approx \Lambda_0/1.8$ [50] as estimate for the cosmological constant. Depending on the ansatz for the metric this should be modified by a corresponding small integer as prefactor.

[A8.2] dS$_4$ and energy density

A relationship to a de Sitter space requires a constant energy density, w(r) = const, a condition not met for w(r) of a particle. However, for any dependence on a type of energy density $w(r) \sim -1/r^N$, e.g. for the square of the electric field $E^2(r)$: $w(r) \sim -1/r^4$, w(r) may for a discrete value r_l be decomposed in $w_1(r) \sim -1/(r-r_l)^4$, and $w_2(r) \sim -1/r_l^4$ = const, with $|w_1(r)| >= |w_2(r)|$, see fig. 10. This allows to separate a constant small w_2 term from any $-1/r^N$ term [51]. A dS$_4$-like component may thus be considered as a general background for any length scale.

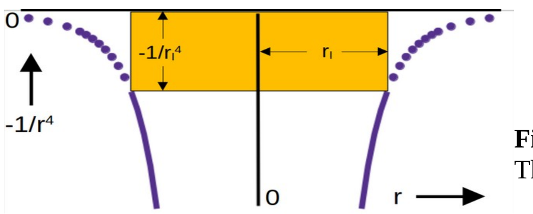


Fig. 10: Decomposition of energy density, w(r); The length scale refers to a radius in 3D-space.

[A9] Numerical calculation

Example for numerical calculation with r = r_e (spin=1/2), r_l (spin=√3/2) and r_{ll} (spin=1) as boundary condition (bold in line 1 for r: multiplicator for r-entries, 1000 steps):

π	3.14159
c_0	3.00E+8[m/s]
σ_0	1.810E+08
ß	4.909E-22
$e_c/(4\pi\varepsilon_c)$	7.419E-11[m]
$e_c^2/(4\pi\varepsilon_c)$	2.307E-028[J/m]
h_bar	1.055E-034[Js]

Δr	'=$((r_n-r_{n-1})+(r_{n+1}-r_n))/2$
ρ_0	'= $e_c/(4\pi\varepsilon_c)$
φ(r)	'=$\exp(-1.5^3\sigma_0ß(\rho_0/r)^3)$
φ*(r)	'=$\exp(-\sigma_0ß(\rho_0/r)^3)$
ΔW(r)[J]	`=$2\varepsilon_c\rho_0^2\varphi(r)\ \Delta r/r^2$
ΔS(r)[Js]	`=$2\pi W(r)r/c_0$

Spin = 1/2

	r	φ	ΔW(r)	ΔS(r)
	1,00783			
	6.00E-16	5.96E-247		
	6.05E-16	2.95E-241	1.75E-255	2.223E-278
	6.09E-16	1.08E-235	6.36E-250	8.124E-273
	6.14E-16	2.93E-230	1.71E-244	2.207E-267
	6.19E-16	5.95E-225	3.46E-239	4.49E-262
	6.24E-16	9.13E-220	5.27E-234	6.884E-257
	6.29E-16	1.06E-214	6.08E-229	8.009E-252
	6.34E-16	9.43E-210	5.36E-224	7.114E-247
	6.39E-16	6.44E-205	3.63E-219	4.855E-242
	6.44E-16	3.40E-200	1.90E-214	2.562E-237
	6.49E-16	1.39E-195	7.73E-210	1.051E-232
	6.54E-16	4.47E-191	2.46E-205	3.373E-228
	6.59E-16	1.13E-186	6.17E-201	8.516E-224
	1.30E-12	1.00E+00	2.76E-18	7.5426E-38
	1.31E-12	1.00E+00	2.74E-18	7.5426E-38
	1.32E-12	1.00E+00	2.72E-18	7.5426E-38
	1.33E-12	1.00E+00	2.70E-18	7.5426E-38
	1.34E-12	1.00E+00	2.68E-18	7.5426E-38
	1.35E-12	1.00E+00	2.66E-18	7.5426E-38
	1.36E-12	1.00E+00	2.64E-18	7.5426E-38
	1.38E-12	1.00E+00	2.62E-18	7.5426E-38
	1.39E-12	1.00E+00	2.60E-18	7.5426E-38
	1.40E-12	1.00E+00	2.58E-18	7.5426E-38
	1.41E-12			
Sum			**8.26E-14**	**5.27E-35**
Target value			8.19E-14	5.27E-35
Ratio			1.009	1.000

Spin = √3/2

	r	φ	ΔW(r)	ΔS(r)	
	1,011865				
	6.00E-16	5.96E-247			
	6.07E-16	2.17E-238	1.94E-252	2.47E-275	
	6.14E-16	3.97E-230	3.51E-244	4.524E-267	
	6.22E-16	3.75E-222	3.28E-236	4.276E-259	
	6.29E-16	1.87E-214	1.62E-228	2.135E-251	
	6.36E-16	5.04E-207	4.31E-221	5.754E-244	
	6.44E-16	7.50E-200	6.34E-214	8.555E-237	
	6.52E-16	6.28E-193	5.25E-207	7.164E-230	
	6.59E-16	3.02E-186	2.49E-200	3.447E-223	
	6.67E-16	8.51E-180	6.95E-194	9.712E-217	
	6.75E-16	1.43E-173	1.15E-187	1.633E-210	
	6.83E-16	1.46E-167	1.17E-181	1.668E-204	
	6.91E-16	9.23E-162	7.27E-176	1.053E-198	
	6.67E-11	1.00E+00	8.16E-20	1.1407E-37	
	6.75E-11	1.00E+00	8.07E-20	1.1407E-37	
	6.83E-11	1.00E+00	7.97E-20	1.1407E-37	
	6.91E-11	1.00E+00	7.88E-20	1.1407E-37	
	6.99E-11	1.00E+00	7.79E-20	1.1407E-37	
	7.07E-11	1.00E+00	7.70E-20	1.1407E-37	
	7.16E-11	1.00E+00	7.61E-20	1.1407E-37	
	7.24E-11	1.00E+00	7.52E-20	1.1407E-37	
	7.33E-11	1.00E+00	7.43E-20	1.1407E-37	
	7.41E-11	1.00E+00	7.34E-20	1.1407E-37	2.7
	7.50E-11				
Sum			**8.30E-14**	**9.11E-35**	
Target value			8.19E-14	9.13E-35	
Ratio			1.013	0.998	

Spin = 1

	r	φ*	ΔW(r)	ΔS(r)
	1,01288			
	6.00E-16	1.11E-73		
	6.08E-16	6.20E-71	6.02E-85	7.6733E-108
	6.16E-16	2.73E-68	2.62E-82	3.3837E-105
	6.23E-16	9.59E-66	9.08E-80	1.1863E-102
	6.32E-16	2.70E-63	2.52E-77	3.3355E-100
	6.40E-16	6.13E-61	5.66E-75	7.58356E-98
	6.48E-16	1.14E-58	1.04E-72	1.40547E-95
	6.56E-16	1.73E-56	1.56E-70	2.13967E-93
	6.65E-16	2.18E-54	1.94E-68	2.69565E-91
	6.73E-16	2.29E-52	2.01E-66	2.83056E-89
	6.82E-16	2.02E-50	1.75E-64	2.49432E-87
	6.91E-16	1.50E-48	1.28E-62	1.85681E-85
	7.00E-16	9.49E-47	8.01E-61	1.17512E-83
	1.79E-10	1.00E+00	3.30E-20	1.23765E-37
	1.81E-10	1.00E+00	3.26E-20	1.23765E-37
	1.84E-10	1.00E+00	3.22E-20	1.23765E-37
	1.86E-10	1.00E+00	3.18E-20	1.23765E-37
	1.88E-10	1.00E+00	3.13E-20	1.23765E-37
	1.91E-10	1.00E+00	3.10E-20	1.23765E-37
	1.93E-10	1.00E+00	3.06E-20	1.23765E-37
	1.96E-10	1.00E+00	3.02E-20	1.23765E-37
	1.98E-10	1.00E+00	2.98E-20	1.23765E-37
	2.01E-10	1.00E+00	2.94E-20	1.23765E-37
	2.03E-10			
Sum			**1.24E-13**	**1.047E-34**
Target value			8.19E-14	1.055E-34
Ratio			1.520	0.993

49 Such as $\rho^3/\mathbf{r}^5$ in [A2.1] though this term cancels in the specific example for G_{00}.

50 With Hubble constant H_0 = 67.66 [km/s/Mpc] $\Lambda_0 \approx$ 1.11E-52 [m^{-2}]; [16]

51 Notably, calculating the energy of such a dS$_4$ for r_l (or r_{ll} - factor $2/3\Gamma_{-1/3}/3$ cancels) according to the boundary condition S= √3ħ/2 (or S=1ħ) yields e_c:

$$W(dS_4) \approx \varepsilon_c\left(\frac{e_c}{4\pi\varepsilon_c r_l^2}\right)^2\int_0^{r_l}\exp\left(-\left(\frac{\rho_0}{r}\right)^3\right)d^3r \approx 3\frac{e_c^2}{(4\pi)^2\varepsilon_c r_l^4}\frac{4\pi r_l^3}{3} \approx e_c$$