Theodor Kaluza's Theory of Everything: revisited

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Abstract

Using a Kaluza-type of model, describing the laws of electromagnetism within the formalism of differential geometry, provides a coherent, comprehensive and quantitative description of phenomena related to particles, including a convergent series of quantized particle energies, with limits given by the energy values of the electron and the Higgs vacuum expectation value, and the values for electroweak coupling constants. The geometry of the solutions for spin 1/2 defines 6 lepton-like and 6 quark-like objects and allows to calculate their fractional electric charges as well as magnetic moments of baryons.

A series expansion links electromagnetic terms, gravitational terms and a de Sitter background that can be related to a cosmological constant and an a_0 parameter of MOND/BTFR in the correct order of magnitude.

The model can be expressed *ab initio*, necessary input parameters are the electromagnetic constants.

1 Introduction

Theodor Kaluza in 1919 developed a unified theory of gravitation and electromagnetism that produced the formalism for the field equations of the general theory of relativity (GR) and Maxwell's equations of electromagnetism (EM) thus unifying the major forces known at his time. His 5-dimensional model [1] is not suited to give properties related to particles, a problem addressed by Oskar Klein [2] who introduced the idea of compactified extra dimensions and attempted to join the model with the emerging principles of quantum mechanics. Therefore the theory is mainly known as Kaluza-Klein theory today and in this version became a progenitor of string theory. The work presented here does not follow this path but rather the extension of Kaluza’s original work put forth by Wesson and coworkers [3], [4], a version known as space-time-matter theory. In this concept one makes use of Campbells theorem which states that any curved N-D space can be embedded at least locally in a flat (N+1)-D-(i.e. Minkowski-) space. Using either 4D-curved or 5D-flat space-time is merely a mathematical choice, there is nothing “extra” in terms of physics. This is analog to the approach for a 4D-de-Sitter space (dS_4) which represents a submanifold of the 5D-space-time discussed in the Kaluza context and provides a useful link between particle and cosmological phenomena.

A crucial *simplification* of Kaluza’s original ansatz concerns the constant of gravitation, which is introduced in his metric as a coefficient to render the EM potentials dimensionless. This is a rather unfitting combination and an unnecessary assumption. In this modification G will be omitted [1] as well as all non-EM terms. Gravitation can quantitatively be recovered by a series expansion and G can be expressed as an EM-term.

Curvature of space-time based on an electromagnetic version of the field equations of GR will be strong enough to localize a photon in a self-trapping kind of mechanism and in combination with a boundary condition, spin 1/2, will yield accurate energy states in the range of the particle zoo. Circular polarized light is part of conventional electromagnetic theory, in the following such a feature will be treated equivalently with the terms angular momentum or spin [2] as intrinsic property of particles. In particular, unless noted otherwise, it is assumed that particles posses spin 1/2 or are composed of spin 1/2 components. Spin will be a necessary boundary condition to determine an integration limit for the equations used. Since at this point there is no obvious ansatz for integrating spin into the metric of this model, any metric discussed in the following should be considered as an approximation only [3].

The basic proceeding will be as follows:

Following [4] Kaluza’s equations will be elaborated for flat 5D-space-time. They may be arranged to give [cf. 4, chapter 6.6]:

1) Einstein-like equations for space-time curved by electromagnetic and scalar fields (equ. (3)),
2) Maxwell equations where the source depends on the scalar field,
3) a wave-like equation connecting the scalar Φ with the electromagnetic tensor (equ. (4)).

Solutions of 3) for Φ in a flat 5D-metric will be used in a general ansatz for a metric. Due to 3) Φ includes a

1 In general EM coefficients should be expected. In the electrostatic approximation used here one can do without any coefficient.

2 “Spin” will be used as a generic term not necessarily implying specific features of the quantum mechanical term.

3 dS_4 may cover some aspects of spin, but does not give a relationship to S_z=1/2.

term with an exponential function of the EM-potentials, in the approximation of this work the electric potential, A_{el}. The only other coefficient entering the equations will be ħ of the boundary condition spin 1/2[ħ]. These coefficients are related by the fine-structure constant, α, and Φ can in general be given as function of A_{el} and α, see chpt. 2.4 [4]. The α-terms in the exponential part of Φ may be given as two coefficients: σ, representing an integration limit required for $S_Z = 1/2$ and α_{Pl}, a ground state coefficient for charged particles that can be derived from spin-related considerations. Since a geometric interpretation allows to give α in terms of Γ-functions, chpt 2.8, the results of this model can be calculated *ab initio,* using electromagnetic and mathematical constants only.

The exponential allows to integrate a function r^{-N} and yields an energy in the range expected for neutrinos when inserting σ only and a set of converging series of particle energies as function of α [5] with limits given by the energy of the electron and the Higgs vacuum expectation value (VEV) when inserting both σ and α_{Pl}.

Omitting the spin coefficient σ in a 2^{nd} order term of a series expansion of the exponential leaves the ground state coefficient α_{Pl}. Thus α_{Pl}^{-1} will be the maximum coefficient for a particle beyond the electron allowed for not exceeding the 1^{st}, the EM-term. Within the accuracy of the calculations of this model coefficient α_{Pl} - as calculated from spin related particle parameters! - is identical with the ratio of the electron and Planck energy, (cf. chpt. 2.5, 4.1) and the 2^{nd} order term can be identified with gravitation, as expectable.

The 3^{rd} order term yields values for the cosmological constant, Λ, the Baryonic-Tully-Fisher-Relation (BTFR) of galaxy rotation curves and the related coefficient a_0 of MOND in the correct order of magnitude and may be interpreted in the context of a dS_4 (cf. chpt. 4.3).

Focusing on the angular momentum aspects of the model, in chpt. 3 the rotation of a set of orthogonal E, B, C-vectors, attributed to the electromagnetic fields and the propagation with the speed of light, C, will be modelled via quaternions. This gives 3 possible solutions for spin 1/2 defining 6 distinct geometric objects with partial charges of 1/3 and 2/3. Combining 2 complementary solutions gives 6 lepton like entities as the simplest, node-free case, combining 3 appropriate solutions allows to calculate magnetic moments of baryons.

Typical accuracy of the calculations is in the order of 0.0001 [6]. The deviation of calculated results from the experimental particle values is typically in the range 0.01 - 0.001, consistent with a variation of input parameters related to elementary charge in an order of magnitude of QED corrections, which are not included in this model.

To focus on the more fundamental relationships some minor aspects of the model are exiled to an appendix, related topics will be marked as [A].

1.1 System of natural units

In the following a unit system based on SI for units of mechanics but with modified EM-units, indicated by subscript c, will be used that is particularly suited for simplifying the relevant expressions [7]:

$$c_0^2 = (\varepsilon_c\ \mu_c)^{-1} \qquad (1)$$

with $\varepsilon_c = (2.998E+8\ [m^2/Jm])^{-1} = (2.998E+8)^{-1}\ [J/m]$

$\mu_c = (2.998E+8\ [Jm/s^2])^{-1} = (2.998E+8)^{-1}\ [s^2/Jm]$.

From the Coulomb term $b_0 = e^2/(4\pi\varepsilon_0) = e_c^2/(4\pi\varepsilon_c) = 2.307E-28$ [Jm] follows for the square of the elementary charge: $e_c^2 = 9.671E-36\ [J^2]$. In the following $e_c = 3.110E-18$ [J] and $r_l = e_c/(4\pi\varepsilon_c) = 7.419E-11$ [m] may be used as natural unit of energy and length [8].

2 Calculation of energies and coupling constants

2.1 Kaluza theory

Kaluza theory is an extension of general relativity to 5D-space-time with a metric given as [cf. 4, equ. 2.2]:

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4 The coefficient of angular momentum may be interpreted as either σ, which will in general indicate the integration limit, $(r/\rho)^3$ for calculating the incomplete gamma functions or its main component $\alpha_{lim} \approx 1.5/\alpha$, see chpt. 2.4, 2.5.

5 The relation of the masses e, μ, π with α was noted first in 1952 by Nambu [5]. MacGregor calculated particle mass and constituent quark mass as *multiples* of α and related parameters [6].

6 Including e.g. errors due to the numerical approximation of incomplete Γ-functions.

7 In SI proper ρ_0, ρ have units of [Vm]; coefficient δ [1/V] would be part of the argument of the exponential.

8 This is the only unique choice for a unit system with energy units for charge and is preferred over picking an arbitrary coefficient, such as elementary charge, electric constant, energy of the electron, etc. as additional constant of nature. With this choice both e_c and r_l correspond to specific coefficients of the electron related to spin, cf. chpt. 2.4.2, 4.3.2.

$$g_{AB} = \begin{bmatrix} (g_{\alpha\beta} - \delta^2 \Phi^2 A_\alpha A_\beta) & -\delta \Phi^2 A_\alpha \\ -\delta \Phi^2 A_\beta & -\Phi^2 \end{bmatrix} \quad (2)$$

In (2) roman letters correspond to 5D, Greek letters to 4D. δ corresponds to a general constant appropriate for an EM unit system that turns δA_α into a dimensionless quantity. To get the field equations of GR Kaluza assigns δ to a gravitational term. Assuming 5D space-time to be flat, i.e. $G_{AB} = 0$, gives for the 4D-part of the field equations [cf. 4, equ. 2.3]:

$$G_{\alpha\beta} = \frac{\delta^2 \Phi^2}{2} T^{EM}_{\alpha\beta} - \frac{1}{\Phi}\left(\nabla_\alpha (\partial_\alpha \Phi) - g_{\alpha\beta} \Box \Phi\right) \quad (3)$$

From $R_{44} = 0$ follows:

$$\Box \Phi = -\frac{\delta^2 \Phi^3}{4} F_{\alpha\beta} F^{\alpha\beta} \quad (4)$$

2.2 Modification of Kaluza’s metric

In the following the focus will be on EM-terms only, i.e. $g_{\alpha\beta}$ of (2) is set to zero. The model is further restricted to consider only the electric potential, $A_{el} = e_c/(4\pi\varepsilon_c r) = \rho_0/r$ [-]. In the following ρ_0 will refer to the A_{el}-term, while dropping subscript 0 will indicate a general solution where ρ may contain additional terms.

This is an approximation not only in neglecting contributions of the magnetic potentials but also in not considering spin. It is therefore not possible to give an *exact* metric for the problems considered here. [A2] introduces 2 versions for illustrative purposes. The following approximations will be used.

Only derivatives with respect to r of a spherical symmetric coordinate system will be considered.

According to Campbells theorem [4] a flat (N+1)-D metric is mathematically equivalent to a curved N-D metric so both approaches are compatible in principle. Solutions of (4) for an approximate Φ of a flat 5D-metric will be used as general ansatz in a 4D-metric. In

$$\Phi_N \approx \left(\frac{\rho}{r}\right)^{N-1} \exp\left(-\left(\frac{\rho}{r}\right)^{N} / 2\right) \quad (5)$$

the term of highest order of exponential N, given by $\Phi'' \sim \rho^{3N-1}/r^{3N+1}$, may be interpreted to provide the terms for $A_{el}(r)' \sim e_c/(4\pi\varepsilon_c r^2) \sim \rho_0/r^2$, see [A1]. The significance of (5) lies in providing the relationship of exponential and pre-exponential terms and first of all in the requirement to contain powers of (ρ_0/r) in the exponent of Φ_N.

The difference in order of magnitude between ρ_0 and ρ, to be elaborated on below, results in the leading term for particle energy being due to the Christoffel symbols of the angular coordinates [9], yielding a solution for particle energy that is essentially independent from minor details of the metric, including the use of either a 4D- or a 5D-metric.

Concerning dimensions and unit systems:

Since E is the derivative of a unitless δA_{el}, $(\delta E)^2$ will have appropriate units for T_{00} (with 1.1: $E^2[1/m^2]$). A term with energy density w in T_{00} would require an expansion with some appropriate coefficient for an electric constant, ε, turning the square of the electric field into energy density, $(\varepsilon\delta^2E^2)$, which in turn requires ε to cancel in T_{00}: $T_{00} = 1/\varepsilon\, (\varepsilon\delta^2E^2) = 1/\varepsilon\, w$.

When equating G_{00} with T_{00} the G-term in the conventional field equation will be replaced by 1/ε, giving

$$(8\pi) G/c_0^4 \quad \Rightarrow \quad \approx \quad 1/\varepsilon \quad (6)$$

in

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = -\frac{1}{\varepsilon} T_{\alpha\beta} \quad (7)$$

The relation between the gravitational and the EM scale will be given by a numerical factor of

$$\gamma_{CG} = 8\pi\varepsilon_c G/c_0^4 = 6.93\text{ E-}52 \quad (8)$$ [10]

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9 Giving a “-1” component in the Ricci-tensor; This is consistent with curvature being due to the lateral extension of the E-vector in the quaternion ansatz of chpt. 3;

10 Note: γ_{CG} is closely related with α_{Pl} of (28): $\gamma_{CG}/2 = \alpha_{Pl}^2 (e_c/W_{electron})^2 = (e_c/W_{Planck})^2$.

2.3 Point charge energy

A broad class of metrices will give (9) as solution for G_{00} (cf.[A2]):

$$G_{00} = - \rho_0^2/r^4 \exp(-(\rho/r)^3) = w/\varepsilon_c \tag{9}$$

The exponential of function Φ allows to integrate the electric field of the point charge. The volume integral over the energy density of (9) gives the energy of particle n according to:

$$W_n = \varepsilon_c \rho_0^2 \int_0^{r_n} \exp(-(\rho_n/r)^3) r^{-4} \, d^3r = 4\pi \varepsilon_c \rho_0^2 \int_0^{r_n} \exp(-(\rho_n/r)^3) r^{-2} \, dr \tag{10}$$

Solutions for integrals over $\exp(-(\rho/r)^3)$ times some function of r can be given by:

$$\int_0^{r_n} \exp(-(\rho_n/r)^N) r^{-(m+1)} dr = \Gamma(m/N, (\rho_n/r_n)^N) \frac{\rho_n^{-m}}{N} = \int_{(\rho_n/r_n)^N}^{\infty} t^{\frac{m}{N}-1} e^{-t} dt \, \frac{\rho_n^{-m}}{N} \tag{11}$$

in this work used for N = {3; 4}, m = {-2; -1; 0; +1;+2}. The term $\Gamma(m/N, (\rho_n/r_n)^N)$ denotes the upper incomplete gamma function, given by the Euler integral of the second kind [11]. In the range of values relevant in this work, for $m/N \geq 1$ the complete gamma function $\Gamma_{m/N}$ is a sufficient approximation, for $m/N \leq 0$ the integrals have to be calculated numerically, requiring an integration limit, see 2.4.
Equation (10)f will give:

$$W_{n,elstat} = 4\pi \varepsilon_c \rho_0^2 \int_0^{r_n} \exp(-(\rho_n/r)^3) r^{-2} \, dr = b_0 \Gamma(+1/3, (\rho_n/r_n)^3) \rho_n^{-1}/3 \approx b_0 \Gamma_{+1/3} \rho_n^{-1}/3 \tag{12}$$

Particles are supposed to be electromagnetic objects possessing photon-like properties, thus it will be assumed that particle energy, W_n, has equal contributions of electric and magnetic energy, i.e.:

$$W_n = W_{n,elstat} + W_{n,mag} = 2W_{n,elstat} \approx 2 b_0 \Gamma_{+1/3} \rho_n^{-1}/3 \tag{13}$$

2.4 Angular momentum, coefficient σ

2.4.1 $S_Z = 1/2$ [ħ]

The integral limits required for integrals of (11) with $m/N \leq 0$ are r_n („particle radius" of state n; with respect to S_Z [12]) for integrals over $\exp(-(\rho/r)^3)$ and $(\rho_n/r_n)^3$ for the Euler integrals. The latter will be expressed via a constant defined as 8/σ [13]:

$$(\rho_n/r_n)^3 = 8/\sigma \tag{14}$$

whose value may be derived from the condition for angular momentum $S_z = 1/2$ [ħ]. A simple relation with angular momentum S_z for spherical symmetric states will be given by applying a semi-classical approach [14]:

$$S_z = r_2 x \, p(r_1) = r_2 W_n(r_1)/c_0 \equiv 1/2 \, [\hbar] \tag{15}$$

Using term $2b_0$ of equ. (13) as constant factor and integrating over a circular path of radius $|r_2| = |r_1|$, equation (11) will give for m = 0:

$$S_z = \int_0^{r_n}\int_0^{2\pi} S_z(r,\varphi) d\varphi \, dr = 4\pi \frac{b_0}{c_0} \int_0^{r_n} e^{-\left(\frac{\rho}{r}\right)^3} r^{-1} dr = 4\pi \alpha\hbar \int_0^{r_n} e^{-\left(\frac{\rho}{r}\right)^3} = \frac{4\pi}{3} \alpha\hbar \int_{8/\sigma}^{\infty} t^{-1} e^{-t} dt$$
$$= \frac{4\pi}{3} \alpha\hbar \Gamma(0, (\rho_n/r_n)^3) \equiv \frac{[\hbar]}{2} \tag{16}$$

To obtain $S_z = 1/2$ [ħ] the integral over $\exp(-(\rho/r)^3)/r \, dr$ of (16), *has to* yield $\alpha^{-1}/8\pi$.

$$\int_0^{r_n} \exp(-(\rho_n/r)^3) r^{-1} dr = 1/3 \int_{8/\sigma}^{\infty} t^{-1} e^{-t} dt \equiv \frac{\alpha^{-1}}{8\pi} \approx 5.45 \tag{17}$$

Relation (17) may be used for a numerical calculation of the integration limit, 8/σ, giving a value of σ_0

11 Euler integrals yield positive values, the sign convention of Γ-functions gives negative values for negative arguments. Abbreviations such as $\Gamma_{-1/3}$ for $|\Gamma(-1/3)|$ will be used;

12 $r_n = \lambda_{C,n}/\sqrt{3}$ see (75);

13 Chosen to give coefficient σ as component in the argument of the exponential of Φ, see [A3.1].

14 In 0^{th} approximation: using the term for energy (13) and length (21)f requires $\sigma^{1/3}$ to be of order of the inverse fine-structure constant α^{-1}: $1/c_0 \int w(r)\, dr * \int dr \approx b_0/\rho_n * \sigma^{1/3}\rho_n/c_0 \equiv \hbar/2 \Rightarrow \sigma^{1/3} \approx \alpha^{-1}$.

(assumed to represent spherical symmetry), σ_0 = 1.810E+8 [-]. Assuming the coefficient $\Gamma_{-1/3}/3$ according to (11) has to be part of the expression for σ_0 [15] this results in $\sigma_0 \approx 8(1.5\alpha^{-1}\Gamma_{-1/3}/3)^3$. This value may be interpreted as a coefficient representing geometry, giving a value close to the numerical one:

$$\sigma_0 \approx 8\,(1.5\,\alpha^{-1}\,\Gamma_{-1/3}/3)^3 \approx 8\left(\frac{4\pi\Gamma_{-1/3}{}^3}{3}\right)^3 = 1.772\text{E+8 [-]} \tag{18}$$

As a consequence a dimensionless volume-like term appears in the denominator of the energy expression (13) for spherical symmetry. Expression (18) will be used for illustrative purposes and to gain additional insights into the relations of this model, e.g. in deriving a geometric approximation for α in chpt. 2.8.

In [A3] some additional aspects of the terms supposed to constitute σ will be discussed, giving an alternate expression for (18) and demonstrating that coefficient σ has to be part of the exponent of Φ:

$$\rho(\sigma)^3 \approx \sigma\rho_0^3 \tag{19}$$

Calculating energy according to ρ(σ) and (13) will give W in the order of magnitude of 0.1 eV, a value in the estimated energy range for a neutrino, see tab.1.

For particles associated with charge (or partial charges) an additional coefficient will be needed. For the lowest energy state of the electron this will be denoted α_{Pl} and $\alpha_{Pl} < 1$ has to hold to yield an energy value larger than that of a pure point charge state, to be discussed in 2.5.

$$\rho(\sigma, \alpha_{Pl})^3 \approx \sigma_0\alpha_{Pl}\,\rho_0^3 \tag{20}$$

2.4.2 Calculation of "radius", S = √3/2 [ħ]

The results of 2.4.1 with σ_0, r_n referring to S_Z = 1/2 [ħ] are assumed to represent particle states, i.e. localized EM-objects of a certain stability. There exists a second characteristic length scale, r_l, related to S = √3/2 [ħ] that represents a transition point to the free point charge.

2.4.2.1 Particle "radius"

In the following a term for length expressed via the Euler integral of (11) will be introduced for a general length r_x:

$$r_x = \int_0^{r_x} e^{-\left(\frac{\rho}{r}\right)^3} dr = \rho_n/3 \int_{(\rho_n/r_x)^3}^{\infty} t^{-4/3} e^{-t}\, dt \approx \Gamma(-1/3, (\rho_n/r_x)^3)\ \rho_n/3 \tag{21}$$

In the limit $(\rho_x/r_x)^N$ -> 0

$$\Gamma(-1/N, (\rho_x/r_x)^N) = \int_{(\rho_x/r_x)^N}^{\infty} t^{-(1/N+1)} e^{-t}\, dt \approx N\,(\rho_x/r_x)^{-1} = N\,\sigma_x^{1/3}/2 \tag{22}$$

holds (last term from (14)). The 3rd term of equation (22) inserted in the right side of (21) gives back r_x, however, the relations of (21)f may be seen as expressing r_x in terms useful for this model, i.e. ρ_n, σ_0 and Γ-functions.

2.4.2.2 Extremal value for "radius"

Increasing the integral limit r_x results in increasing $\Gamma(-1/3\,(\rho_x/r_x)^3)$ and σ_x. Thus at a particular value of r_x the product of σ_x and the extra terms in ρ compared to ρ_0 ($[1.5\sigma_0^{1/3}\alpha_{Pl}^{1/3}]$, cf. (29)) cancel, i.e. $\sigma_x[1.5\sigma_0^{1/3}\alpha_{Pl}^{1/3}] = 1$ and $r_x = r_l = \rho_0 = e_c/(4\pi\varepsilon_c)$ holds. This is a unique point where the coefficients representing particles will be cancelled in the expression for r (equ. (21), (29)):

$$r_l \approx r_e\frac{r_l}{r_e} = \frac{\sigma_0^{1/3}}{2}[1.5\sigma_0^{1/3}\alpha_{Pl}^{1/3}\rho_0]\,\frac{r_l}{r_e} \approx \rho_0 \tag{23}$$

Terms in brackets refer to ρ_e. Within the accuracy of this model r_l will coincide with a value for angular momentum of S = √3/2 [ħ] which marks a characteristic limit for the existence of particles. The value of r_l allows to calculate α_{Pl}, cf. 2.5.3 and will be of crucial importance for the series expansion of chpt. 4. The ratio r_l/r_e can be approximated by $\approx 2/3^{1.5}\,\alpha^{-1}$.

2.4.3 Lower limit of σ

The minimal possible value for σ is defined by the Γ-term in the integral expression for length, $\Gamma_{-1/3}/3$, (11), and the integers in (67):

15 Since according to (14) $\sigma^{1/3}$ is proportional to a length parameter, r_n, which according to (11) includes $\Gamma_{-1/3}/3$.

$\sigma_{min} = 8(\Gamma_{-1/3}/3)^3$ [16] (24)

leaving a term

$$\alpha_{\lim} \approx 1.5\alpha^{-1} \approx 4\pi\Gamma_{-1/3}^{\;2} \tag{25}$$

as variable part in σ to consider non-spherical symmetric states (see 2.6, [A3]) [17]). According to α_{lim} the maximum additional contribution to W_{max} with respect to a spherical symmetric state would be:

$$\Delta W_{max} \approx 3/2\ \alpha^{-1}\ . \tag{26}$$

2.5 Quantization with powers of $1/3^n$ over α

Most relations given here are valid for any particle energy which should be expected as there is a continuous spectrum of energies according to special relativity. However, a particular set of energies may be identified by relaxing the condition of orthogonality of different states according to quantum mechanics to requiring different states to be expressible in simple terms of a ground state coefficient (α_0, α_{Pl}) in the exponent of Φ and not to exhibit any dependence on intermediary states [18].

There are 3 lines of thought for an estimation/calculation of α_0, α_{Pl}.

2.5.1 Energy relative to a pure point charge

The condition that energy/length of a charged particle has to be higher/smaller than the value given by a pure electrostatic term.

Since r_n according to (13), (18)f will be proportional to $\approx \alpha^{-2}$ the term in the exponential has to be: $\alpha_0 < (\alpha^2)^3$. The relationship between a photon-like object and a point charge object of elementary charge is based on the coefficient α, suggesting a photon-like state to differ by an additional factor of α from a pure point charge state and to use a ground state coefficient $\alpha_0 \approx \alpha^9$. This fits the relationship of a set of fundamental particle energies with the charged particle of lowest energy, the electron, as a ground state quite well, however, requiring an *ad hoc* factor ≈ 3/2 for the electron itself [20]. With W_e as ground state, W_n would be given by (27)ff *relative* to the electron state as (see table 1):

$$W_n/W_e \approx 3/2\frac{\alpha\wedge(1.5/3^n)}{\alpha^{1.5}} \approx 3/2\,\Pi_{k=1}^{n}\alpha\wedge(-3/3^k) \qquad n = \{1;2;..\} \tag{27}$$ [21]

However, to calculate *absolute* values of energy requires another factor in addition to α_0.

2.5.2 Gravitation as second order term

In a series expansion of the exponential in terms of force, potential, etc., such as given below, chpt. 4, particles beyond the electron enter the terms according to their coefficients from (27). In order for the 2nd order term not to exceed the 1st order term the energy of spherical symmetric particles - including relativistic effects - should not exceed $\alpha_0^{-1} = \alpha^{-9}$. However, this restriction should apply for non-spherical symmetric particles as well, requiring $\alpha_{lim} \approx 1.5/\alpha$ as additional factor. Including the additional factor of the electron, ≈ 3/2, and restricting to electrostatic contributions (i.e. factor 2 in the denominator might correspond to relate only the electrostatic contributions of (13) for the electron with the electrostatically defined value of a Planck state, cf. equ.(41)) gives:

$$\frac{1.5^3\alpha_0}{2\alpha_{\lim}} \approx 1.5^2\alpha^{10}/2 \approx 4.8E\text{-}22 \equiv \alpha_{Pl} \approx \frac{W_e}{W_{Pl}} \tag{28}$$

The electron coefficient in the exponential of Φ and the energy term equ. (13) would be given as: $\alpha_e \approx (3/2)^3 \alpha^9$. ρ_n may be given as (δ = 1 for electron, = 0 otherwise; n = {0;1;2;..}):

$$\rho_n^3 \approx (1.5^3)^\delta\, \sigma_0\, \boldsymbol{\alpha_{lim}^{-1}/2\ 1.5^3\, \alpha^9}\, \alpha^{4.5}/\alpha\wedge(4.5/3^n)\ \ (e_c/(4\pi\varepsilon_c))^3 \approx (1.5^3)^\delta\, \sigma_0\, \boldsymbol{\alpha_{Pl}}\, \alpha^{4.5}/\alpha\wedge(4.5/3^n)\ \ (e_c/(4\pi\varepsilon_c))^3 \tag{29}$$

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16 For N = 3 (22) and (24) give: $\Gamma(-1/3, (\rho/3)^3) = 3/2\ 2\Gamma_{-1/3}/3 = \Gamma_{-1/3}$; If the term of (18) is interpreted as a (cube of a) volume parameter, a term of the kind of (24) might represent the (cube of a) 1D parameter.

17 $\sigma_0 \approx (\alpha_{lim}\, 2\Gamma_{-1/3}/3)^3 \approx (\Gamma_{-1/3}/\alpha)^3$

18 cf. $W_n^2 \sim \left(\alpha_0^{1/3}\alpha_0^{1/9}....\alpha_0^{1/(3\wedge(n-1))}\boldsymbol{\alpha_0^{1/(3\wedge n)}}\right) / \left(\boldsymbol{\alpha_0^1}\alpha_0^{1/3}\alpha_0^{1/9}....\alpha_0^{1/(3\wedge(n-1))}\right) = \boldsymbol{\alpha_0^{1/(3\wedge n)}/\alpha_0}$. The 3rd power relationship might be related to Lorentz boosts and spin, see [A3.3].

20 Factor 1.5 is used for simplicity, most calculations indicate that a slightly higher value of 1.51x is more appropriate. This is discussed in [A3.2].

21 A supposed neutrino state according to (19) roughly fits such a 3rd power partial product as well: $W_n/W_v \approx \Pi^n_{k=0}\ \alpha\wedge(-3/3^k)$ n={0;1;..}.

2.5.3 Direct calculation from S = √3/2

The unique value for radius, r_l, defined by S = √3/2 [ħ] in chpt. 2.4.2.2 allows to calculate α_{Pl} directly by solving (23) for α_{Pl}:

$$\alpha_{Pl} \approx \left(\frac{r_e}{r_l}\right)^3 \frac{2^6}{3^3}\sigma_0^{-2} \tag{30}$$

and fitting for r_l, S = √3/2 or both. Depending on which parameters one sets the focus on and the preferred approach for σ_0, accuracy may vary in a range of ~ 1%, typical for this model.

In the common depiction of S as axial vector representing a cone around an axis in z-direction the transition S_z -> S involves closing this cone until S itself is directed in z-direction. In the quaternion ansatz of chpt. 3 this is analogous to the closing of the cone of the E-vector, implying vanishing angular momentum and B-field, two features necessary for the transition to the field of a point charge.

A value S = √3/2 [ħ] in this sense has no equivalent in the QM interpretation and moreover rather represents a limit for vanishing angular momentum. In this Kaluza model it marks the limit of a particle state:

- it is the value where particle specific coefficients cancel in the exponent of Φ, leaving the pure electric potential term,
- it is the necessary limit for a series expansion of the exponential of Φ to yield a gravitation term, chpt. 4.2,
- it is related to the upper limit of the particle energy series, cf. [A7.3].

2.6 Non-spherical symmetric states

Assuming the angular part to be related to spherical harmonics and exhibiting the corresponding nodes would give the analog of an atomic p-state for the 1st angular state, y_1^0. With the additional assumption that $W_{n,l} \sim 1/r_{n,l} \sim 1/V_{n,l}^{1/3} \sim (2l+1)^{1/3}$ ($V_{n,l}$ = volume) is applicable for non-spherically symmetric states this would give $W_1^0/W_0^0 = 3^{1/3} = 1.44$ and $\sigma^{1/3} = 3^{-1/3}\,\sigma_0^{1/3}$. The considerations of chpt. 3.1 support that a y_1^0-like symmetry for particle states has to be considered and a second partial product series of energies in addition to (27) corresponding to these values approximately fits the data, see tab. 1 [22].

A change in angular momentum has to be expected for a transition from spherical symmetric states, y_0^0, to y_1^0 which is actually observed with ΔJ = ± 1 except for the pair μ/π with Δ J = 1/2.

With $\sigma^{1/3}/2 = \{4\pi\Gamma_{-1/3}^{\;3}/3\ (y_0^0);\ \ 3^{-1/3}4\pi\Gamma_{-1/3}^{\;3}/3\ (y_1^0);\ \Gamma_{-1/3}/3\ (\text{max})\}$ energy relative to the electron state may be given as:

$$W_n/W_e \approx 3/2\frac{\alpha\wedge\left(1.5/3^n\right)}{\alpha^{1.5}}\frac{\sigma_0^{1/3}}{\sigma^{1/3}} = 3/2\,\Pi_{k=1}^{n}\alpha\wedge(\text{-}3/3^k)\,\frac{\sigma_0^{1/3}}{\sigma^{1/3}} \qquad n = \{1;2;..\} \tag{31}$$

The total maximum energy will be $W_{max} \approx W_e\, 9/4\, \alpha^{-2.5}$ = 4.05E-8 [J] (= 1.03 Higgs vacuum expectation value, VEV = 246GeV = 3.941E-8 [J] [7]) [23].

2.7 Results of energy calculation

Table 1 presents the results of the energy calculation according to (13), (29) for y_0^0 (bold), y_1^0. Only states given in [7] as 4-star, characterized as „*Existence certain, properties at least fairly well explored*", are included, up to Σ'^0 all such states given in [7] are listed. Coefficients given in col. 4 refer to (27), (29), starting with the electron coefficient in W_e, including its extra term of 2/3. Exponents of -9/2 for Δ and tau are equal to the limit of the partial product of α(n), including the electron coefficient. The term [f(l)] represents $\sigma_0^{1/3}/\sigma^{1/3}$ in equ. (31).

In col. 5 equ. (13) and (29) are used to calculate energy with σ_0 according to the value of the *fit for S_Z =1/2 and α_{Pl} given by W_e/W_{Pl} according to the experimental value* of the electron and definition (41) for Planck energy.

$$W_n = 2b_0\int_0^{r_n}\exp\left(-\left(1.5^{3\,\delta}\sigma_0\,\alpha_{Pl}\frac{\boldsymbol{\alpha}^{\mathbf{4.5}}}{\boldsymbol{\alpha}^{(\mathbf{4.5/3^n})}}\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right)\right)r^{-2}dr \quad => \quad W_\mu = \frac{2}{3}\frac{\Gamma_{+1/3}\,\boldsymbol{\alpha}^{-\mathbf{1}}}{\left(\sigma_0\alpha_{Pl}\right)^{1/3}}\;e_c \tag{32}$$

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22 Considerations such as given in [A7] may give some indication why a 1:1 correspondence between y_1^0-like states and mesons should not be expected.

23 For the Higgs boson see [A7.3].

(n = {0;1;2;..}; 1.5^{δ} = extra coefficient for the electron only, δ = δ(0,n); bold: particle coefficient; muon given as example[24]). In col. 6 an alternate version - attempting to replace the values from fit and experiment by a more fundamental though speculative ansatz - according to (74) of [A3.3] is given for comparison. Additional particle states and blanks in the table are discussed in [A7]. The values of physical constants are taken from [7].

To illustrate possible QED-effects and the non-linearity of the Γ-functions, a calculation of σ_0 with values of (16)f varying within +/-1.00116 gives a range of energy values of +/-1.006, varying within $+/-1.00116^2$ gives a range of energy values of +/-1.013 compared to the values given in table 1 [25]. Additional effects due to e.g. different charge in particle pairs of same isospin have to be expected.

The accuracy of ~1% of the values calculated for leptons, mesons and baryons is comparable to that of LQCD calculations for baryons [8].

	n, l	$W_{n,Lit}$ [MeV]	α-coefficient in W_n $\alpha(n)^{-1/3}$ [f(l)]	W_{calc}/W_{lit} Equ.(32)	W_{calc}/W_{Lit} Equ.(74)	J
ν	**-1***	**'~ E-7**	**0**	-		-
e^{+-}	**0, 0**	**0.51**	**$2/3\ \alpha^{-3}$**	**1.014**	**1.002**	**1/2**
μ^{+-}	**1, 0**	**105.66**	**$\alpha^{-3}\alpha^{-1}$**	**1.008**	**0.996**	**1/2**
π^{+-}	1, 1	139.57	$\alpha^{-3}\alpha^{-1}$ [$3^{1/3}$]	1.101	1.088	0
K		495	[A6]			0
η^0	**2, 0**	**547.86**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}$**	**1.002**	**0.990**	**0**
ρ^0	2, 1	775.26	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3})$ [$3^{1/3}$]	1.022	1.009	1
ω^0	2, 1	782.65	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3})$ [$3^{1/3}$]	1.012	1.000	1
K*		894	[A6]			1
p^{+-}	**3, 0**	**938.27**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**1.011**	**0.999**	**1/2**
n	**3, 0**	**939.57**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**1.010**	**0.998**	**1/2**
η'		958	[A6]			0
Φ^0		1019	[A6]			1
Λ^0	**4, 0**	**1115.68**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}$**	**1.020**	**1.008**	**1/2**
Σ^0	**5, 0**	**1192.62**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81}$**	**1.014**	**1.002**	**1/2**
Δ	**∞, 0**	**1232.00**	**$\alpha^{-9/2}$**	**1.012**	**1.000**	**3/2**
Ξ		1318				1/2
Σ^{*0}	3, 1	1383.70	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9})$ [$3^{1/3}$]	0.989	0.977	3/2
Ω^-	4, 1	1672.45	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27})$ [$3^{1/3}$]	0.982	0.970	3/2
N(1720)	5, 1	1720.00	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81})$ [$3^{1/3}$]	1.014	1.002	3/2
tau^{+-}	∞, 1	1776.82	$(\alpha^{-9/2})$ [$3^{1/3}$]	1.012	1.000	1/2
Higgs	∞,∞ **	1.25 E+5	$(\alpha^{-9/2})$ [3/2 α^{-1}] /2	1.024	1.012	0
VEV	∞,∞**	2.46 E+5	$(\alpha^{-9/2})$ [3/2 α^{-1}]	1.042	1.029	

Table 1: Particle energies; col.2: radial, angular quantum number; col.4: α-coefficient in W_n according to (32), n = {0;1;2;..}; col.5,6: ratio of calculated energy, W_{calc} according to (32), (74)) and literature value [7] (* see (19), ** chpt. 2.6, [A7.3]); col.7: angular momentum S_z [ħ];

2.8 Fine-structure constant, α

Using equ. (22) for the incomplete Γ-function and multiplying r_x in the integration limit $(\rho_n/r_x)^3$ by √3 to obtain a term for Compton wavelength, $\lambda_{C,n}$, (see [A4, (75)]), gives in good approximation (using (18)):

$$\lambda_{C,n} \approx 3^{1.5}\ \sigma_0^{1/3}/2\ \rho_n/3 \approx 3^{0.5}\ 4\pi\ \Gamma_{-1/3}^{3}/3\ \rho_n \tag{33}$$

With (33) energy of a photon may be expressed as:

$$W_{Phot,n} = hc_0/\lambda_{C,n} = hc_0 \Big/ \int^{\lambda_{C,n}} e^{-\left(\frac{\rho}{r}\right)^3} dr = \frac{2\,hc_0}{3^{0.5}\rho_n\sigma_0^{1/3}} \approx \frac{3\,hc_0}{3^{0.5}\,4\pi\,\Gamma_{-1/3}^3\rho_n} \tag{34}$$

24 The term for the muon is given as reference to avoid ambiguities due to extra term ≈ 3/2 of the electron.

25 This involves Γ(0,x); the nonlinearity of Γ-functions in the parameter range of this model increases Γ(+1/3,x) << Γ(0,x) << Γ(-1/3,x).

Assuming the energy of a particle to be the same in both photon and point charge description. Equating (13) with (34) gives:

$$W_{pc,n} = W_{Phot,n} = 2b_0\, \Gamma_{+1/3}\, \rho_n^{-1}/3 \approx \frac{3\, hc_0}{3^{0.5}\, 4\pi\, \Gamma_{-1/3}^3\, \rho_n} \quad (35)$$

Solving equ. (35) for α will involve a term of two Γ-functions with an argument of same value and opposite sign for which the relation Γ(+x)Γ(-x) = π /(x sin(πx) holds [9], giving for the product $\Gamma_{+1/3}\Gamma_{-1/3}$:

$$\Gamma_{+1/3}\,\Gamma_{-1/3} = 3^{0.5}\, 2\pi \quad (36)$$

Using equation (35) with (36) will give

$$\alpha^{-1} = \frac{hc_0}{2\pi b_0} \approx \left(\frac{2\Gamma_{+1/3}}{3^{0.5}\, 2\pi}\right)\left(\frac{4\pi}{3}\Gamma_{-1/3}^3\right) \approx \frac{2}{3}\frac{\Gamma_{-1/3}}{\Gamma_{+1/3}}\, 4\pi\, \Gamma_{+1/3}\Gamma_{-1/3} \approx \boldsymbol{4\pi\, \Gamma_{+1/3}\Gamma_{-1/3}} \quad (37)$$

The last expression is emphasized since it has a simple interpretation in terms of the coefficients of the integrals over $\exp(-(\rho/r)^N)$. Equations (35)ff are based on the integral over a 3-dimensional point charge term modified by the exponential term according to (5) with N = 3, and a complementary integral - in 3D for length, λ_C - to yield a dimensionless constant. This may be generalized to N dimensions (N ={3; 4}), to give a point charge term (S_N = geometric factor for N-dimensional surface, in case of 3D: 4π; 4D: $2\pi^2$):

$$\int_0^r \exp(-(\rho/r)^N)\, r^{-2(N-1)}\, d^N r = S_N \int_0^r \exp(-(\rho/r)^N)\, r^{-(N-1)}\, dr \quad (38)$$

that has to be multiplied by a complementary integral

$$\int_0^r \exp(-(\rho_n/r)^N)\, r^{(N-3)}\, dr \quad (39)$$

The exact result depends on the integration limit of the second integral, cf. [A4]. However, in terms of the Γ-functions both electroweak coupling constants can be given in 1st approximation as

$$\alpha_N^{-1} = S_N\, \frac{\Gamma(+m/N)\,\Gamma(-m/N)}{m^2} = S_N\, \frac{\Gamma(+(N-2)/N)\,\Gamma(-(N-2)/N)}{(N-2)^2} \qquad \text{(m = N-2, cf. (11))} \quad (40)$$

Constant	Calculated values of *inverse* of coupling constant, α_N^{-1}, weak mixing angle		Exp. values
α_{weak}	$2\pi^2\, \Gamma_{+1/2}\, \Gamma_{-1/2}/4 = \pi^3 =$	31.0	30.4-31.7
α	$4\pi\, \Gamma_{+1/3}\, \Gamma_{-1/3} = 4\pi\, \Gamma_{+1/3}\, \Gamma_{-1/3} =$	136.8	137.036
$\sin^2(\theta_W)$	α/α_{weak}	0.227	0.222-0.231
$\cos(\theta_W)$	m(W-boson)/m(Z-boson)	0.879	0.882

Table 2: Results for 1st approximation of electroweak coefficients [26]

3 Quaternion ansatz

3.1 Basic approach

The model as described above emphasizes a Kaluza-like ansatz with spin as boundary condition. Reversing the main focus, emphasizing angular momentum and implicitly assuming curvature of space as necessary boundary condition for localization is a straight forward alternate way to get additional information about the states of this model, details are given in [A5].

A circular polarized photon with its intrinsic angular momentum interpreted as having its E- and B-vectors rotating around a central axis of propagation, C, will be transformed into an object of SO(3)-type symmetry where the center of rotation is the origin of a triple of EBC-vectors, supposed to be locally orthogonal [27]. This has the following qualitative consequences:

1) Such a rotation is related to the group *SO(3)* and *SU(2)* as important special case. In the following a quaternion ansatz will be used for modelling the respective rotations.

26 Experimental values: PDG [7]: $\sin^2\theta_W = 0.231$, CODATA [10]: $\sin^2\theta_W = 0.222$).

27 In the limit r -> λ_C => |C| -> $|c_0|$ (flat space-time); In place of C the Poynting vector may be used;

2) E-vector constantly oriented to a fixed point implies *charge*. As implicitly assumed above, neutral particles are supposed to exhibit nodes separating corresponding equal volume elements of reversed E-vector orientation and opposite polarity.
3) A local coordinate system = rest frame implies *mass*.
4) In case of any lateral extension of the E-field, for r -> 0 the overlap of a rotating E-vector implies rising energy density, resulting in *rising curvature of space-time* according to GR or its modification as of equ. (7).
5) The EBC-triple can be given in 2 different *chiral* states (left- right-handed).
6) As essentially electromagnetic waves such states are consistent with a "point-like" structure function on the other hand imply a spatial distribution of energy density and angular momentum / spin.
7) Antiparticles may be constructed by switching orientation of fields and chirality.

For quantitative results 3 orthonormal vectors E, B, C, each described as imaginary part of a quaternion with real part 0, will be subject to alternate, incremental rotations around the axes E, B and C. In the following only solutions where one of the incremental angles of rotation has half the value of the other two will be considered. This may serve as a primitive model for spin $S_Z = 1/2$. There are 3 possible solutions corresponding to half the angular velocity for each of the components E, B, or C. The trajectory of the E-vector encloses a spherical cone, the spherical cap of the cone encompasses a fraction of the area of a hemisphere of 2/3, 1/3 and 1/3, respectively. Mirroring at the center of rotation gives the equivalent double cone (dark grey in fig.1), the fractions of both caps in relation to the surface of the total sphere may be interpreted to give partial charges of 2/3, 1/3 and 1/3 according to Gauss' law. In the following such components will be assigned to uds-quark-like entities, the assignment (half-frequency-E-rotation, charge +2/3, U), (half-B, charge -1/3, D), (half-C, charge -1/3, S) will be used.

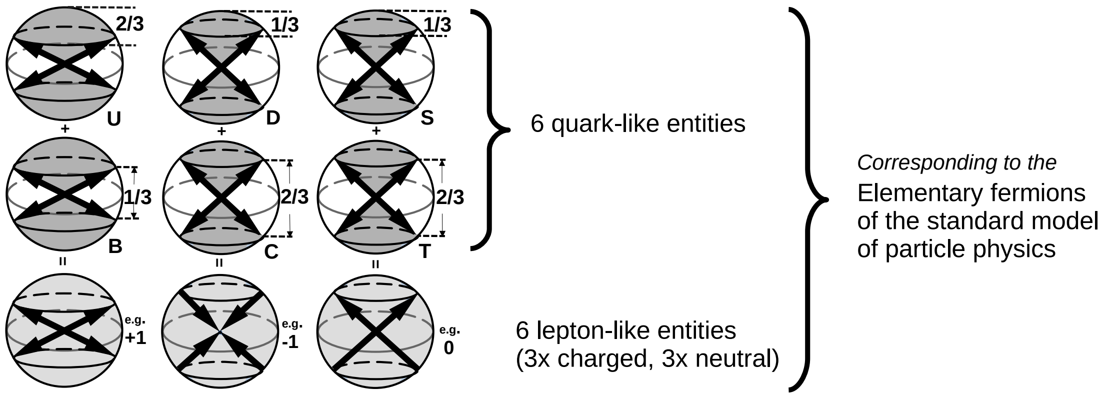

Fig.1: Trajectories of the E-vector, enclosing spherical cones and spherical wedges

The E-vector might as well be interpreted to enclose the complement of the double cone of a 3D-ball (white in fig.1), a spherical wedge. This gives the objects complement-U, complement-D, complement-S with charges 1/3, 2/3, 2/3. These objects may be assigned to cbt-quark-like entities.
A combination of two cones to give a double cone will always give a valid solution with any spin or chirality and may be considered to correspond to the y_1^0 solutions of chpt. 2.6 [28].
The simplest combination of 2 entities (grey + white) of fig.1 will consist of 2 complementary segments of same charge, chirality, phase, etc., to recover a simple sphere with *no nodal planes* (last row of fig. 1). Such particles should represent the lowest possible energy state, *S_Z = 1/2 should still be valid* and charge could have values of +/-1 or 0. An electron might be considered e.g. as an (anti-U + (U-Complement = B)) particle, however, unlike a B-meson with spin 1/2. While this is not possible with quarks, i.e. objects with particle character, it would represent the simplest solution for such a type of an electromagnetic wave.
The neutral configuration will have to be distinct from all other particles by representing a state where the center of rotation is not at the "tip" of an E-vector, but at its "center", see last row right in figure 1. This will be an intrinsically neutral particle unlike particles consisting of components of opposite charge, such as the neutron, and a unique solution that for geometric reasons is not suited as component to build other particles. It will not be subject to the conditions of 2.5 and α_{Pl}, which are related to "charge".

28 Composite objects - in particular if composed of 3 UDS-components – may feature sufficient spherical symmetry to conform to the respective energy equation (32). The spherical symmetry of nucleons as assumed in chpt. 2 may be given by a suitable linear combinations of the states discussed in [A5], cf. [A7.2, η].

3.2 Magnetic moments of baryons

A particular sensitive test for such a toy model will be the calculation of the magnetic moments of baryons from an orthogonal combination of 3 U,D,S-units (cf.[A5.2]). It is possible to give values for all combinations of the uds-octet of spin 1/2 that match the experiment within a few percent, however, they have to be selected from a larger set of solutions. Unique solutions require additional boundary conditions, for nucleons this will be isospin: exchanging U- and D-components results in switching the values for magnetic moment of p and n, (cf.[A5.3]).

	$\lvert M\rvert Exp[Am^2]$	$\lvert M\rvert Calc[Am^2]$	$\lvert M\rvert Calc/ \lvert M\rvert Exp$
p^{+-}	1.41E-26	1.39E-26	0.988
n	9.66E-27	9.55E-27	0.988
p^+/n			1.001187

Table 3: Magnetic moments for proton and neutron (units in standard SI, cf. [A5.2]); |M|exp: [7];

A simple analysis for particles with S-components is not possible due to differences in symmetry (cf. tab. 4 in [A5]) that prevent a simple cancelling of S_z=1/2 components by exchange of U-S- or D-S-units.

3.3 Chirality / "Color"

The orthonormal EBC-vectors feature two possible chiral configurations, right-handed "R" and left-handed "L", suggesting to be a possible source for a factor 3 frequently appearing in the quantitative interpretation of processes involving a quark-antiquark-pair, such as in the decay of the W- or Z-boson or in the coefficient R of electron-positron-annihilation. While this is attributed to the 3 "colors" of quarks in the SM, the same factor would result for any pair of quark-like states having the possibility to exist in triplet-like states, "LL", "RR" and (1/√2) (LR+RL) [29] (referring to an axial vector representing the EBC-configuration).

4 Higher order phenomena - gravitation, de Sitter space

4.1 Planck scale

In this work the expression

$$b_0 = G\, m_{Pl}^2 = G\, W_{Pl}^2 / c_0^4 \tag{41}$$

is used as definition for Planck terms, giving for the Planck energy, W_{Pl}:

$$W_{Pl} = c_0^2 (b_0 /G)^{0.5} = c_0^2 (\alpha\hbar c_0/G)^{0.5} = 1.671\ E{+}8\ [J] \tag{42}$$

The value of W_{Pl} according to definition (42) allows to identify the ratio of W_e and W_{Pl} with the α-terms given in (28), i.e. the relation between W_e and W_{Pl} is given by $\alpha_e \approx (3/2)^3 \alpha^9$, the electron coefficient in the exponential part of Φ, divided by two times the factor α_{lim} for non-spherical symmetric contributions according to (25). The constant G may be given as:

$$G \approx \frac{\alpha_{Pl}^2 c_0^4 b_0}{W_e^2} \tag{43}$$

Since α_{Pl} and W_e may be expressed as function of π, $\Gamma_{+1/3}$, $\Gamma_{-1/3}$ and e_c, (18), (28), (32)/(74) and (37), G may be expressed as a coefficient based on EM constants only, $G \approx 2/3\ c_0^4\alpha^{24}/(4\pi\varepsilon_c) = 6.6799E{-}11\ [m^3/(kgs^2)]$.

4.2 Gravitation from series expansion of exponential function

Terms for gravitation may be recovered via a series expansion of either $\Gamma(+1/3, (\rho_n/r_n)^3)$ [30] of (12) or the exponential part of Φ in any suitable expression, e.g. potential $e_c/(4\pi\varepsilon_c r)$, resulting in a general term such as:

$$\frac{e_c}{4\pi\varepsilon_c r}\left[1 - \sigma\alpha_{Pl}\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right] \approx \text{Coulomb-term}\left[1 - \sigma\alpha_{Pl}\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right] \tag{44}$$

which is a very good approximation for $r > \alpha\lambda_C$. The 1st term is the classic Coulomb term, the 2nd term contains by definition (cf. (28), (41)) the ratio between Coulomb and gravitational terms for *one* electron, α_{Pl}. To turn this into the exact Coulomb / gravitation relationship requires:

1) coefficient σ to approach unity,
2) parameter r in $e_c /(4\pi\varepsilon_c r)$ to turn into a constant,

29 With a singlet state corresponding to destructive interference;

30 $\Gamma(1/3, (\rho_n/r)^3) \approx \Gamma_{1/3} - 3(\rho_n/r) + 3/4(\rho_n/r)^4 - 3/7(\rho_n/r)^7 + \ldots$ [9]

3) parameter r to approach the value, $r_l = e_c/(4\pi\varepsilon_c)$.

=>1) coefficient σ is essentially related to spin of a particle and it has to be assumed that spin does not play a role for $r \geq r_n, r_l$ (supported by considerations of 2.5.3),

=> 2) r *in the exponential* may not be a free parameter for $r \geq r_n, r_l$, the approximate limit of a real solution for an equation such as (67),

=> 3) will be met if the integration limit according to the approach of chpt. 2.4.2, 2.5.3 is set to give a value for angular momentum S = √3/2 [ħ].

The general expression for the series expansion would be:

$$\text{Coulomb-term } (1 - \alpha_{Pl} + \alpha_{Pl}^2/2 - ....) \qquad (45)$$

Coefficient α_{Pl}^2 necessary to give the full equivalent for replacing the constant G is evenly split on both particles involved in gravitational interaction, i.e. the second α_{Pl} has to be contributed from the second particle, multiplied by appropriate coefficients from the α-series according to the α(n) and σ coefficients of (31) for particles other than the electrons in a rest frame. Since according to chpt. 2.5.2 the 2nd term of such a series expansion should not exceed the 1st, electromagnetic one, the maximum (relativistic) mass for spherical symmetric particles would be defined by $\alpha_e^{-1} = \alpha^{-9}$ while the inverse of the maximum angular term, i.e. α_{lim}^{-1} as given in (25) secures that particles that are not spherical symmetric in a rest frame can not exceed the Planck limit either. The maximum general, i.e. non rest-frame coefficient allowed for one particle would be α_{Pl}^{-1}.The force of gravitation between two particles 1 and 2 would be given by:

$$F_G = \frac{G m_1 m_2}{r^2} = \alpha_1 \alpha_2 \alpha_{Pl}^2 F_C = \frac{\alpha_1 \alpha_2 \alpha_{Pl}^2 e_c^2}{4\pi\varepsilon_c r} = \frac{\alpha_1 \alpha_2 \gamma_{CG} W_e^2}{8\pi\varepsilon_c r} \quad [31] \qquad (46)$$

In the equations of the next chapters α_{Pl}^2 will be used to indicate the transition from EM to gravitational scale, i.e. as part of a replacement of G, according to (6)ff. In analogy, the 3rd term in the series expansion should feature α_{Pl}^4.

4.3 The 3rd order term of the series expansion

The following will examine some links between Λ / deSitter-space / MOND, based mainly on ideas of Milgrom [11] and Aldrovandi, Pereira, et al. [12], and the results of this modified Kaluza ansatz, in particular the series expansion of chpt. 4.2.

4.3.1 Λ / de Sitter-space / MOND / BTFR

The proportionality constant in the Einstein Field Equations (EFE), $8\pi G/c_0^4$, originates from equating $G_{00} = T_{00}$ in the “weak field limit” (WFL) of low acceleration with the corresponding term from Newton’s law. With a non-zero Λ this relation can not be strictly valid any more and a deviation from the Newton case due to Λ has to be expected [32].

MOND (Modified Newton Dynamics) is an ansatz for the WFL and early on a numerical relationship between the pivotal parameter in MOND, a_0, and Λ as well as a possible relationship with a 4D-de Sitter space (dS_4) has been pointed out [11].

De Sitter space is the maximally symmetric solution of the EFE with a positive constant, Λ. It is characterized by a constant energy density and can be described as a submanifold of a Minkowski space of dimension n+1, for dS_4:

$$\pm x_0^2 + x_1^2 + x_2^2 + x_3^2 + x_4^2 = R_{dS}^2 \qquad (47)$$

R_{dS} is a length parameter, the “radius” of the de Sitter space. The scalar curvature R of de Sitter space is given by

$$R = \frac{n(n-1)}{R_{dS}^2} = \frac{2n}{n-2}\Lambda. \quad \text{(n =dimension of embedded space)} \qquad (48)$$

for dS_4: $\Lambda = 3/R_{dS}^2$.

+/- x_0 represents a time coordinate, the sign defines hyperbolic or spherical symmetry of the dS_4. In the following only symmetry of a 3-sphere for space and great circles as geodesics will be considered, i.e. a

31 In the last term energy refers to W_e in place of e_c, α_{Pl}^2 has to be replaced by γ_{CG} of (8) according to $\gamma_{CG} = (W_e/e_c)^2 \alpha_{Pl}^2$.

32 In a simple ansatz the effects of Λ on gravitational phenomena would be relevant on the scale of intergalactic interaction and Mpc [13]. Araujo et al. consider a local Λ-term as function of energy density, Λ(r) ~ w(r), in “de Sitter modified general relativity” to describe the BTFR [12b].

closed solution (or imaginary time for particles [33]) will be assumed.
In a dS_4-geometry any worldline would formally have an acceleration component a_0':

$$a_0' = v^2/R_{dS} = c_0^2(\Lambda_0/3)^{0.5} \approx 5.46E\text{-}10[m/s^2] \approx 4.55 a_0 \tag{49}$$

One of the best-documented deviations from Newton's limit of GR in the WFL, as expected according to baryonic mass, is represented by the rotation curves of disc galaxies which can be characterized by the Baryonic Tully Fisher Relation (BTFR) [14]:

$$G M_g = v_f^4/a_0 \tag{50}$$

with M_g = total baryonic mass of the galaxy, v_f = "final" rotation velocity for large distances, $a_0 \approx 1.2E\text{-}10$ $[m/s^2]$ = empirical parameter of the BTFR. This corresponds to the so called Deep-MOND regime where $a \ll a_0$. Relations (49) and (50) have a_0 in a reciprocal relationship, the dS_4 interpretation does not fit to MOND in a trivial way.

4.3.2 Λ / de Sitter-space / MOND / BTFR in the modified Kaluza model

A relationship of the expression for the electron with the BTFR may be established as follows:
Looking for an equivalent of (50) for the electron case, using (46), gives with respect to 2 electrons:

$$\frac{G m_e{}^2}{m_e} = G m_e = \frac{\alpha_{Pl}^2}{m_e}\frac{e_c^2}{4\pi\varepsilon_c} = \frac{\alpha_{Pl}^2}{W_e}\frac{c_0^2 e_c^2}{4\pi\varepsilon_c} \quad {}^{34} \tag{51}$$

With an expansion by $\alpha_{Pl}^2 c_0^2$ one gets:

$$G m_e = \frac{\boldsymbol{\alpha_{Pl}^2}}{W_e}\frac{\boldsymbol{c_0^2} e_c^2}{4\pi\varepsilon_c}\frac{\boldsymbol{\alpha_{Pl}^2 c_0^2}}{\alpha_{Pl}^2 c_0^2} = \boldsymbol{\alpha_{Pl}^4 c_0^4}\frac{e_c^2}{\alpha_{Pl}^2 c_0^2 W_{el} 4\pi\varepsilon_c} \approx \frac{\boldsymbol{v_{f,el}^4}}{\boldsymbol{a_0''}} \quad {}^{35} \tag{52}$$

This corresponds to the BTFR with a single electron as source and has a simple interpretation: all coefficients c_0 of the EM scale of the electron [36] are shifted by a factor α_{Pl} resulting in total in α_{Pl}^4 of a 3rd order term. As in the case of gravitation, for other particles the appropriate coefficients according to (31) have to be inserted in (52). To get the BTFR on the scale of a galaxy will require a multiplication by the particle factor of the particles relevant for mass, i.e. 1836 for the nucleons, times the number of nucleons for the particular galaxy, e.g. in case of the Milky Way [37]:

$$a_0'' G M_g = v_f^4 = a_{Pl}^4 c_0^4 * 1836 * 1E{+}68 \approx 9.4E{+}19[m^4/s^4] \quad {}^{38} \tag{53}$$

The resulting velocity of $v_f \approx 100[km/s]$ is in the order of magnitude of the observed value [14].

To express an equivalent term using Λ raises the problem of establishing a suitable relation of R_{dS} to a length parameter of the 3D space. Assuming Λ is related to α_{Pl}^4 would give:

$$\frac{G m_e}{r_{e,x}^2} = \frac{\boldsymbol{\alpha_{Pl}^4 c_0^4}}{a_0'' R_{dS,e,x}^2} \approx \frac{\boldsymbol{c_0^4 \Lambda}}{3 a_0''} \approx \frac{a_0'^2}{a_0''} \tag{54}$$

with a coefficient that would roughly fit to a parameter related to the electron [39], $R_{dS,e,x} \approx r_{e,x} \approx r_e\, e_c/W_e \approx r_{max,e} \approx \sqrt{3}\alpha_{Pl}^2 \Lambda^{-0.5} = (4E\text{-}17[m])^{-1}$.
Equation (54) is an expression that approximately conforms to both (49) and (50) via a 3rd term in the expansion (45) and a 2nd order type relationship to the Newtonian term, with a_0 taking the role of a reference term related to coefficients of the electron shifted by α_{Pl}^2 to the gravitational scale.

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33 Imaginary time might be appropriate for the particles of this model since the exponential function responsible for the expansion of a hyperbolic dS_4 will turn into a periodic function in a spherical dS_4.
34 Using γ_{CG} of (8): $Gm_e = \gamma_{CG} c_o^2 W_e/(8\pi\varepsilon_c)$;
35 $a_0'' \approx 7.7E\text{-}12[m/s^2] \approx a_0/16$;
36 As the exponential of Φ approaches 1 for $r \to r_e$, all EM-coefficients may be considered to approach their flat space values, e.g. c_0. Analog to the BTFR, for particles modelled as a EBC-triple a dS_4 contribution will have to outmatch any other, 4D-geodesic-type contribution to rotational motion at sufficiently large r as well, cf. fig.3.
37 Baryonic mass ≈ 8.8E+10 solar masses [15] => ≈1.8E+41[kg] divided by 1.67E-27[kg] ≈ 1E+68 nucleons;
38 Using the empirical value of $a_0 \approx 16\, a_0''$ => expansion with 16 => $v_f(a_0) \approx 2v_f(a_0'') \approx 200[km/s]$;
39 An average energy density might rather be represented by r_{max}, the maximum of the r-distribution of energy, W(r), than by the particle radius which indicates the distance where W(r) approaches zero.
Using $r_{e,x}$ of (54) to calculate $w_e = \varepsilon_c r_{e,x}^{-2} \approx 2E{+}24\ [J/m^3]$ (EM-scale!) gives roughly the approximate energy density of the electron if referred to r_e according to: $w_e = W_e/r_e^3 \approx 3E{+}22\ [J/m^3]$

A relationship to a de Sitter space requires a constant energy density, w(r) = const, a condition not met for w(r) of a particle. However, the relevant terms do not represent actual w(r) but are shifted by α_{Pl}^2 relative to gravitation, i.e. represent only a tiny fraction of w(r) that does not necessarily has to follow the same kind of w(r)-distribution. This suggests the following interpretation:
For any dependence on a type of energy density w(r) ~ $-1/r^N$, e.g. for the square of the electric field $E^2(r)$: w(r) ~ $-1/r^4$, w(r) may for a discrete value r_l be decomposed in $w_1(r)$ ~ $-1/(r-r_l)^4$, and $w_2(r)$ ~ $-1/r_l^4$ = const, with $w_1(r) >= w_2(r)$, see fig. 2. This allows to separate a constant small w_2 term from any $-1/r^N$ term [40].

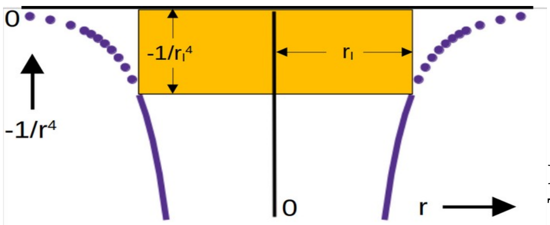

Fig. 2: Decomposition of energy density, w(r);
The length scale refers to a radius in 3D-space.

A dS_4-like component may thus be considered as a general background for any length scale. Trajectories on great circles might be due to small "probe"-objects moving in a dS_4, not to acceleration in 4D-space-time.
Equations (52)f suggest a "local" dS_4 for any finite energy distribution, where a BTFR-like term would be defined by coefficients originating from the electron case times energy, W, relative to energy of the electron, W_e. A specific, "local" Λ(r) would refer to a $R_{dS}(r)$ that is roughly in a similar order of magnitude as the extension of the relevant energy distribution. A limiting "universal" case would correspond to Λ and a maximum energy of $\alpha_{Pl}^{-4}\, W_e \approx$ 1.4E+72J [41]:

$$\alpha_{Pl}^4 \frac{W}{W_e} \frac{1}{R_{dS}(r)^2} \approx \alpha_{Pl}^4 \frac{W}{W_e} \; \Lambda(r) = \Lambda \tag{55}$$

Trajectories on great circles on both length scales should be essentially independent if the scales are sufficiently separated as is certainly the case for the relation universe - galaxy - particle.

A variable Λ(r) will not contradict the 2nd Bianchi identity since Λ(r), including its lower limit Λ, is an integral part of $G_{\alpha ß}$, i.e. it has its origin in the metric, and is separable from the gravitational terms due to its completely different scale and the resulting different physical implications.

Some open questions remain to be examined: Positive Λ is a parameter related to expansion of space-time. Different values of Λ(r) should comply to the established gravitational bound systems e.g. on galaxy scale. As Λ(r) would in general be related to particles this requires to reinterpret the concept of expansion of the universe fundamentally. In (45) the 2nd and 3rd terms have an appropriate opposite sign corresponding to repulsion of the latter. (The analog of the repulsive term (32) in [12b].) The close relation to particles might suggest that expansion could correspond to the expansion of the fields of particles, implying 2 possible scales, EM and gravitational. The concept of vacuum energy of QM/QFT would have to refer to a minimal energy density.

4.3.3 Cosmological constant Λ from minor terms in the metric

A metric of the generic type as described in [A2] will in general produce minor terms that might be considered as a natural candidate for the cosmological constant term, $g_{\alpha\beta}\Lambda$. In [A2.1.2] an example is given to illustrate the emergence of typical extra terms. These will feature the same coefficients as the series expansion and thus might give equivalent terms for Λ directly from the EFE. In particular terms such as $\rho_n^3/\mathbf{r}^5$ or $\rho_n^6/\mathbf{r}^8$ with all r originating from derivatives of the exponential only [42] will yield approximate values in the order of magnitude of critical, vacuum density, ρ_c, ρ_{vac} if setting $r_l = e_c/(4\pi\varepsilon_c)$ as upper bound of r.

$$\frac{\Phi''}{\Phi} \approx \frac{\rho^3}{\mathbf{r}^5} \approx \frac{\alpha_{Pl}}{(e_c/(4\pi\varepsilon_c))^5}\left(\frac{e_c}{4\pi\,\varepsilon_c}\right)^3 = \alpha_{Pl}\;/r_l^2 \;\; = 0.089\ [\mathrm{m}^{-2}] \tag{56}$$

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40 Notably, calculating the energy of such a dS_4 for r_l according to the boundary condition S= √3 ħ/2 yields e_c:

$$W(dS_4) \approx \varepsilon_c \left(\frac{e_c}{4\pi\varepsilon_c r_l^2}\right)^2 \int_0^{r_l} \exp\left(-\left(\frac{\rho}{r}\right)^3\right) d^3r \approx 3\frac{e_c^2}{(4\pi)^2\varepsilon_c r_l^4}\frac{4\pi r_l^3}{3} \approx e_c$$

41 In analogy to gravitation the maximum term that could be inserted in (54)f without exceeding the EM-limit would be $\alpha_{Pl}^{-4} \approx$ 1.7E+85 corresponding to an energy 1.7E+85 $W_e \approx$ 1.4E+72[J].

42 Such as $\rho^3/\mathbf{r}^5$ in [A2.1] though this term cancels in the specific example for G_{00}.

Multiplied by ε_c this gives an energy density of 2.97E-10 [J/m^3], multiplied by $8\pi G/c^4$ this gives for Λ: Λ_{calc}=6.17E-53[m^{-2}] ≈ Λ/1.8 [43] as estimate for the cosmological constant [44].

4.4.3 Summary Λ / de Sitter-space / MOND / BTFR

- a non-zero Λ in the EFE requires a deviation from the Newton limit,
- the value of Λ does not fit phenomena such as BTFR => Λ should be variable => Λ(r), with Λ as limit, which is possible if Λ(r) has its origin in $G_{\mu\nu}$,
- the numerical relationship $a_0 \approx c_0^2/\Lambda^{0.5}$ hints at an interpretation in terms of a dS_4,
- dS_4 features great circles as trajectories,
- a *finite* space with energy density w(r) ~ $1/r^n$ can always be interpreted to have a small de Sitter-like background component,
- the relevant values of Λ and a_0 fit in order of magnitude to an origin in parameters of the electron,
- the first 3 terms in the expansion of the exponential of Φ according to (45) may be interpreted as: electromagnetic + gravitational + dS_4-background (Λ, BTFR, etc.).

5 Discussion

Theory of everything is a somewhat ironic and pompous term and maybe an unachievable goal. At the time Theodor Kaluza’s unification of general relativity and electromagnetism was conceived, it came pretty close, yet the emerging theory of quantum mechanics (QM) moved the finish line. It is a common thought ever since that the theory of General Relativity (GR) somehow has to be unified with QM. The model presented here suggests that the ansatz of Kaluza is sufficient to give an excellent model for particles, in particular in combination with the boundary condition spin 1/2, bypassing QM in 1st approximation [45]. The major deviation from conventional GR is dropping the constant of gravitation in the field equations, a minor thing from a mathematical point of view. The resulting objects of interest are waves only, which naturally fits basic concepts of QM. General features of quantum mechanics that emerge from such an ansatz include quantization of energy or the pivotal constant of quantum mechanics, Planck’s constant, h, that may be derived from the electromagnetic constants and geometry as expressed in the derivation of α.

QM may be seen as an effective theory where the energy density of GR is replaced by a single parameter “mass” and the wave function represents actual wave-like states. Since QM is background dependent and curvature of space-time from the view of the model presented here is not negligible but the dominating effect as far as particles are concerned some concepts of QM might need reconsideration.

Comparing with the quantum field theory (QFT) of the standard model of particle physics (SM):

The results of the quaternion ansatz of chpt. 3 reproduce the set of elementary fermions of the SM. The number of 12 basic building blocks of matter can be traced back to the 3 possibilities to single out one of the orthogonal EBC-vectors and in a broad sense is a consequence of the 3 space dimensions in 4D space-time. While the properties of quarks, such as partial charges, are deduced from experimental particle data they can be *derived* in the quaternion ansatz. Leptons are an integral part of the particle classification scheme.

There are several features of the model that indicate a close relationship with electroweak theory. In addition to the obvious common root in EM there are: SU(2) symmetry, the energy of the Higgs boson /VEV as upper limit for particle energy and the possibility to calculate the IR-limits of the electroweak coupling constants. As for chirality the inherent chiral character of a circular polarized EM-wave is transferred via the orthogonal EBC-triple of the quaternion ansatz to particles.

On the other hand, there seems to be no deeper connection with the concepts of quantum chromodynamics (QCD), such as color [46] or gluons. Properties such as confinement or the need for adhering to the Pauli principle in e.g. the Δ^{++} are obsolete for an object that is an electromagnetic wave. The development of the SM from constituent quarks towards QCD, based on valence and sea quarks plus gluons, was in part required by the limitations in explaining some scattering experiments with 3 point-like objects only. The waves of this model are consistent with a point like structure function and still feature spatial extension.

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43 With Hubble constant H_0 = 67.66 [km/s/Mpc] Λ ≈ 1.11E-52 [m^{-2}]; [16]

44 The expression for Λ_{calc} would correspond to the 3rd order interpretation of chpt. 4.3.2, if r_l^2 is replaced by $R_{dS,e,x} \approx r_{e,x}$, γ_{CG} by α_{Pl}^2 with an additional α_{Pl} (from a second component as in gravitation) being accountable for the difference.

45 QED terms are considered to be a necessary correction for the results of this model.

46 Whose role in the production of quark-antiquark pairs is replaced by chiral pairs, see chpt. 3.3.

Thus not all details of the SM are reproduced by the particle model presented here. However, the relevant benchmark is the agreement with experiments and as for the aspects examined up to now and described above this modified Kaluza model tends to exceed the capabilities of the SM considerably, last not least in regard of the number of free parameters needed: zero. Preliminary results for additional properties such as particle decay or scattering seem to be promising as well (see e.g. [A6]).

The origin in the formalism of GR is a particular strong point since it allows to use the same concepts and parameters from the particle to the cosmological scale. A fundamental model for elementary mass should be expected to yield some information about mass interaction. The only feature added to standard GR and EM is spin and in chapter 2.5 and 4 it is demonstrated that this is the decisive component in determining the ratio of the strength of electrostatic and gravitational interaction.

The examination of a possible relation with phenomena commonly attributed to dark energy and dark matter is in a rather preliminary stage. 4D-de Sitter space, a subspace of flat 5D-Minkowski space-time, seems to be the appropriate concept to link particles, Λ, MOND/BTFR and Kaluza’s 5D-space-time.

The modified Kaluza model provides a fairly comprehensive framework for a wide range of phenomena in physics, yet many details still need significant improvement. Last not least a final version for the metric is missing that includes spin and the magnetic potentials.

Conclusion

A formalism based on a Kaluza ansatz with spin as boundary condition provides a simple, coherent, comprehensive and first of all quantitative description of phenomena related to particles, such as

- a convergent series of particle energies quantized as a function of the fine-structure constant, α, spanning the range from the energy of the electron to the Higgs VEV,
- a single expression for the values of electroweak coupling constants,
- magnetic moments of baryons.

3D-space and spin 1/2 define a set of 6 lepton-like and 6 quark-like objects with the associated charges.

The first 3 terms of the series expansion for Kaluza’s scalar Φ may be interpreted as representing:
electromagnetic + gravitational + dS_4-background - contributions.
The dS_4-background can be related to values of Λ and a_0 of MOND/BTFR in the correct order of magnitude.

The model works *ab initio* without free parameter and allows to remove some values from the set of fundamental constants:
electromagnetic constants, h, G, α, α_{weak}, energies of elementary particles => electromagnetic constants.

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Appendix

In the following the exponential part of Φ^2 is abbreviated as e^v.

[A1] Scalar potential Φ

Equ. (5) may in general be interpreted to refer to the highest order terms of exponential N in Φ":

$$\Phi_N'' \sim \left(\frac{\rho^{3N-1}}{r^{3N+1}}\right)e^{v/2} \sim \boldsymbol{\Phi_N^3}\; e^{-v}\left(\boldsymbol{A_{el}}'\right)^{\mathbf{2}} \approx \left[\left(\frac{\boldsymbol{\rho}}{\boldsymbol{r}}\right)^{\boldsymbol{N-1}}\boldsymbol{e^{v/2}}\right]^{\mathbf{3}} e^{-v}\left(\frac{\boldsymbol{\rho}}{\boldsymbol{r^2}}\right)^{\mathbf{2}} = \left(\frac{\rho}{r}\right)^{3N-3} e^{v/2}\left(\frac{\rho}{r^2}\right)^2 \tag{57}$$

The solutions for the scalar Φ depend on the complete metric used. The easiest method to get a solution of order N is to use spherical coordinates of dimension N+1. Using e.g. the line element for a 4D metric of [4, equ. 6.76]

$$ds^2 = e^v dt^2 - e^\lambda dr^2 - e^\mu r^2\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right) \tag{58}$$

and $A_\alpha = (A_{el}, 0, 0, 0)$ gives as solution for equ.(4) (cf. [4, equ. 6.77], prime corresponds to derivatives with respect to r):

$$\Phi'' + \left(\frac{v' - \lambda' + 2\mu'}{2} + \frac{2}{r}\right)\Phi' - \frac{1}{2}\Phi^3 e^{-v}\left(A_{el}'\right)^2 = 0 \tag{59}$$

This can be solved with a function of type (5) for N = 2:

$$\Phi_2' = \left[-\left(\frac{\rho}{r^2}\right) + 2\left(\frac{\rho^3}{r^4}\right)\right]e^v \tag{60}$$

and

$$\Phi_2'' = \left[2\left(\frac{\rho}{r^3}\right) - 10\left(\frac{\rho^3}{r^5}\right) + 4\left(\frac{\rho^5}{r^7}\right)\right]e^v \tag{61}$$

The ρ^1 terms cancel in (59), the ρ^3 terms can be eliminated by appropriate choice of v', λ' and μ', a remaining factor in the ρ^5 term could be compensated by assuming a corresponding factor in A_{el}. For N = 3 hyperspherical coordinates with the line element

$$ds^2 = e^v dt^2 - e^\lambda dr^2 - e^\mu r^2\left(d\psi^2 + \sin^2\psi\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right)\right) \tag{62}$$

may be used. A more complex metric of the kind given in [A2] may be used as well to solve equation (59).

[A2.1] Metric / point charge

[A2.1.1] Metric with equation (9) as solution

Equ. (9)f. would be the result of the following metric (which does not account for spin!):

$$\begin{aligned} g_{\mu\mu} &= \left(\frac{\rho_0}{r}\right)^2 \exp\left(-\left(\frac{\rho}{r}\right)^3\right)A_{el}^2,\; -\left(\frac{\rho_0}{r}\right)^2\exp\left(\left(\frac{\rho}{r}\right)^3\right)A_{el}^2,\; -r^2 A_{el}^2,\; -r^2\sin^2\vartheta\, A_{el}^2 = \\ g_{\mu\mu} &= \left(\frac{\rho_0}{r}\right)^4 \exp\left(-\left(\frac{\rho}{r}\right)^3\right),\; -\left(\frac{\rho_0}{r}\right)^4\exp\left(\left(\frac{\rho}{r}\right)^3\right),\; -\rho_0^2,\; -\rho_0^2\sin^2\vartheta \end{aligned} \tag{63}$$

[A2.1.2] Metric with typical extra terms

The following gives an alternate metric in some detail to illustrate the significance and order of magnitude of the relevant terms:

$$g_{\alpha\alpha} = \left(\frac{\rho_0}{r}\right)^2 \exp\left(-\left(\frac{\rho}{\mathbf{r}}\right)^3\right), \; - \left(\frac{\rho_0}{r}\right)^2 \exp\left(\left(\frac{\rho}{\mathbf{r}}\right)^3\right), \; -r^2, \; -r^2 \sin^2\vartheta \tag{64}$$

The variable r is marked bold if originating from the exponential term to facilitate a discussion of the implications of its restricted range of validity.

$\Gamma_{01}{}^0 = \Gamma_{10}{}^0 = -1/r^1 + 3/2\, \rho^3/\mathbf{r}^4$ $\quad\Gamma_{00}{}^1 = -1/r^1 e^{-2v} + 3/2\, \rho^3/\mathbf{r}^4 e^{-2v}$

$\Gamma_{11}{}^1 = -1/r^1 - 3/2\, \rho^3/\mathbf{r}^4$

$\Gamma_{12}{}^2 = \Gamma_{21}{}^2 = \Gamma_{13}{}^3 = \Gamma_{31}{}^3 = +1/r^1$ $\quad\Gamma_{22}{}^1 = -r^3/\rho_0{}^2 e^{-v} = \Gamma_{33}{}^1/\sin^2\vartheta$

$\Gamma_{23}{}^3 = \Gamma_{32}{}^3 = \cot\vartheta$ $\quad\Gamma_{33}{}^2 = -\sin\vartheta\cos\vartheta$

$R_{00} = e^{2v} [+1/r^2 + 6\, \rho^3/\mathbf{r}^5 - 9/2\, \rho^6/\mathbf{r}^8)]$

$R_{11} = +3/r^2 - 6\, \rho^3/\, \mathbf{r}^5 + 9/2\rho^6/\mathbf{r}^8$

$R_{22} = -1 + e^{+v} [+ r^2/\rho_0{}^2 + 3\rho^3 r^3/(\rho_0{}^2\mathbf{r}^4)]$

$R = +2/r^2 + e^v [(-4/\rho_0{}^2 - 6\rho^3 r/(\rho_0{}^2\mathbf{r}^4) + 12\, a\, \rho^3\, r^2/(\rho_0{}^2\, \mathbf{r}^5) - 9\, a^2\, \rho^6\, r^2/(\rho_0{}^2\, \mathbf{r}^8)]$

$G_{00} = e^{2v} [+1/r^2 + 6\rho^3/\mathbf{r}^5 - 9/2\rho^6/\mathbf{r}^8)] - e^v \rho_0{}^2/r^4 + e^{2v} [2/r^2 + 3\rho^3/(r\mathbf{r}^4) - 6\, a\, \rho^3/\, \mathbf{r}^5 + 9/2\, \rho^6/(\rho_0{}^2\, \mathbf{r}^8)] =$

$- e^v \rho_0{}^2/r^4 + e^{2v} [3/r^2 + 3\rho^3/(r\mathbf{r}^4)]$

Volume integrals over any ρ^n/r^{n+2} term will yield energy results $\varepsilon_c\int e^v\, \rho^n/r^{n+2}\, d^3r \approx \varepsilon_c\, \rho \approx$ 1E-22 [J] compared to the term $\varepsilon_c\int e^v\, \rho_0{}^2/r^4\, d^3r \approx \varepsilon_c\, \rho_0{}^2\, \rho^{-1} \approx$ 1E-13 [J] (both with coefficients for the electron, $\sigma_0\alpha_{Pl}$) giving negligible contributions to particle energy within the parameter range discussed here. This leaves the first term as leading order: $G_{00} = -e^v\, \rho_0{}^2/r^4$.

[A2.2] General solution N = {1; 2; 3}

This article has a focus on a solution of (5) with N = 3. However, all solutions in a 5D space-time according to [A1], i.e. up to using hyperspherical coordinates, N = {1; 2; 3}, might be used for the ansatz of a metric such as

$$g_{00} = \sum_{N=1}^{3} \left(\frac{\rho_0}{r}\right)^{N-1} \exp\left(-\left(\frac{\rho}{r}\right)^N\right) \tag{65}$$

With the approximation $\sigma \approx 1$ and assuming an identical coefficient α_{Pl} in each term this gives for g_{00}:

$$g_{00} = \exp\left(-\alpha_{Pl}\left(\frac{\rho_0}{\mathbf{r}}\right)\right) + \left(\frac{\rho_0}{r}\right)\exp\left(-\alpha_{Pl}\left(\frac{\rho_0}{\mathbf{r}}\right)^2\right) + \left(\frac{\rho_0}{r}\right)^2\exp\left(-\alpha_{Pl}\left(\frac{\rho_0}{\mathbf{r}}\right)^3\right) \tag{66}$$

Each term might be expanded and split in EM and gravitational part as suggested in chpt. 4.2.

The 3rd term corresponds to the case discussed above, resulting in terms giving the square of the E-field in G_{00} and eventually particle energy as a kind of self energy as well as an equivalent term for gravitation from the series expansion. The second term is the linear version and might be used to construct solutions for potential terms. The first term might represent a general vacuum solution, i.e. without presence of any field ρ_0/r.

[A3] Model coefficients

[A3.1] Coefficient σ as component in ρ

The exponential term, $\exp(-\rho^3/r^3)$, together with the r^{-2} dependence of the field of a point charge define a maximum of particle energy near $r_{W(max)} \approx \rho$, rapidly approaching 0 for $r_{W(max)} > \rho$, effectively allowing to approximate particle energy without using a specific upper integration limit, r_n, see fig. 3. On the other hand the weaker r-dependence of angular momentum, ~1/r results in the calculated values being completely dominated by an integration limit. The limit of the Euler integral of a particle is given by $\rho_n{}^3/r_n{}^3$, a constant which will be denoted $8/\sigma$ in this work.

A general exponential function of radius featuring a limit radius, assumed to correspond to a damped oscillator-like solution and a discriminant term, may be given in 1st approximation as:

$$e^{v'} = \exp\left(-\left(\frac{ß\rho'^3}{2r^3} + \left[\left(\frac{ß\rho'^3}{2r^3}\right)^2 - 4\frac{ß\rho'^3}{2r^3}\right]^{0.5}\right)\right) \tag{67}$$

ß being some general coefficient. At the limit r_n of the real solution of (67)

$$\left(ß\rho'^3/r_n^3\right)^2 = 8ß\rho'^3/r_n^3 \quad => \quad ß = 8\left(\frac{r}{\rho'}\right)^3 = \sigma \tag{68}$$

holds, reproducing the definition of σ according to (14). Within the parameter range of this work for calculating particle energy the function $e^{v'} \approx \exp(-(ß\rho'^3/r^3))$ is a very good approximation of an equation of the kind of (67) and coefficient σ will have to be part of the exponential.

For numerical calculations using a term of type (67) and $r > \alpha r_n$, i.e. limits to calculate S_z or S, requires an additional factor to appear in the denominator of the linear term of the discriminant ($\approx \sigma_0$ in case of S_z).

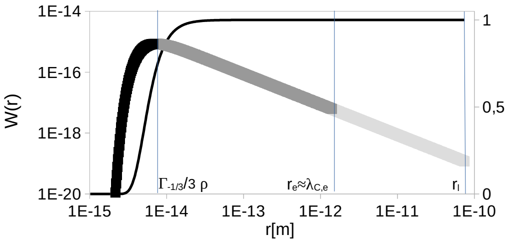


Fig. 3: W(r) for electron, indicating the 3 characteristic length scales $\Gamma_{-1/3}/3\,\rho_e \approx W(r) = \max.$, r_e ($S_z = 1/2$ [ħ]), and r_l ($S = \sqrt{3}/2$ [ħ]); thin black line: $\exp(-(\rho/r)^3)$;
Corresponding energies (numerical calc.):
$\int W(0\text{->}\Gamma_{-1/3}/3\,\rho)/W_e = 0.238$;
$\int W(0\text{->}r_e)/W_e = 0.991$;
$\int W(0\text{->}r_l)/W_e = 0.995$;

[A3.2] Coefficient σ, coefficient 1.5x

The basic relation of $\alpha(n)$ and σ with the fine-structure constant α and coefficient $\Gamma_{-1/3}/3$ is due to the considerations of chpt. 2.4ff. To get a more detailed description in a range of 1% precision is difficult since there are several options conceivable and in this range of accuracy, QED and other minor effects may be expected, which might be amplified due to the non-linear nature of the Γ-functions involved. A factor ≈ 3/2 appears in several terms such as $\sigma_0 \sim 1.5\alpha^{-1}$ of (18), the ratio of electron and muon energy =1.5088, $\Gamma_{-1/3}/\Gamma_{+1/3}$ =1.516, π/2 = 1.5707 and the irregular electron coefficient in the power series that is part of α_{Pl} as well. The following discusses some relevant aspects with a focus on identifying possible underlying relationships while minimizing assumptions about the term ≈ 3/2 in particular.
To get the value of e_c from (12)f coefficient Γ(+1/3)/3 is required to appear as a term in $W(e_c)$ due to the Euler integral, thus a counter term has to be part of ρ in (12):

$$W(e_c) = \frac{e_c^2}{4\pi\varepsilon_c}\int \exp\left(\frac{-\Gamma_{+1/3}}{3}\frac{e_c}{4\pi\varepsilon_c}\right)^3 r^{-2}dr = \frac{e_c^2}{4\pi\varepsilon_c}\frac{\Gamma_{+1/3}}{3}\left(\frac{\Gamma_{+1/3}}{3}\frac{e_c}{4\pi\varepsilon_c}\right)^{-1} = e_c \qquad (69)$$

To deal with both $\Gamma_{-1/3}$ and $\Gamma_{+1/3}$ an additional term of 2π in the denominator of ρ and relation (36) might be useful, e.g. in (considering only basic coefficients of the relevant expressions):

$$\lambda_C \sim 3^{0.5}\int \exp-\left(\frac{\Gamma_{+1/3}}{2\pi 3}\frac{e_c}{4\pi\varepsilon_c}\right)^3 dr \sim \frac{\Gamma_{-1/3}\Gamma_{+1/3}}{2\pi 3^{0.5}}\;\frac{e_c}{4\pi\varepsilon_c} = \frac{e_c}{4\pi\varepsilon_c} \qquad (70)$$

With the simplest version of σ, $\sigma_{lim} = (2\Gamma_{-1/3}/3)^3$ an additional term $((2\pi)^{-1}\Gamma_{+1/3}/\Gamma_{-1/3})^3$ (bold in (71)) in ρ would cancel redundant $\Gamma_{-1/3}/3$ terms in the corresponding length expression as well:

$$\lambda_C \sim 3^{0.5}\frac{\sigma_{\lim}^{1/3}}{2}\rho \approx 3^{0.5}\frac{\Gamma_{-1/3}}{3}\frac{2\Gamma_{-1/3}}{3}\;\boldsymbol{\frac{\Gamma_{+1/3}}{2\pi\Gamma_{-1/3}}}\frac{e_c}{4\pi\varepsilon_c} = \frac{2}{3}\frac{e_c}{4\pi\varepsilon_c} \qquad (71)$$

The term 2/3 would be cancelled if 3/2 of the electron (29) would be included in ρ. The term $(\Gamma_{+1/3}/(2\pi\Gamma_{-1/3}))^3$ consists of components related to angular momentum and (with an additional factor 2) seems to be a suitable replacement for $1/(2\alpha_{lim})$ e.g. in (28)f and may thus be used in expressions such as (72)ff .
Using these coefficients considered essential for yielding basic quantities such as e_c, r_l, including the term 2π associated with angular momentum, and attempting to get a term corresponding to the 3rd power structure of the equations best would give for σ_0:

$$\sigma_{0\#} = \left[\frac{1}{4}\left(\frac{\Gamma_{-1/3}2\pi}{\Gamma_{+1/3}}\right)^3\frac{2\Gamma_{-1/3}}{3}\right]^3 = \left[\left(\frac{\Gamma_{-1/3}\pi}{\Gamma_{+1/3}}\right)^3\frac{4\Gamma_{-1/3}}{3}\right]^3 \quad = 2.008\text{E+8 [-]} \;\; (\text{fit: } \sigma_0 = 1.810\text{E+8 [-]}) \qquad (72)$$
[47]

[A3.3] Model calculations for e^v

In col. 6 of tab. 1 equ. (13) and (29) are used with σ_0 according to (72), α_{Pl} will be replaced by $\alpha_{lim}^{-1}/2$ $(3/2\ \alpha^9)$ with α_{lim} being recalculated from $\alpha_{lim}^{-1} = \sigma_{0\#}^{-1/3}\,2\Gamma_{-1/3}/3$. This gives (73) as expression for e^v [48].

$$\exp\left(-\left[\left(\rho_n/r\right)^3\right]\right) \approx \exp\left(-\left[1.5^{3\delta}\sigma_{0\#}\alpha_{Pl}\alpha(n+1)\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right]\right) \approx \exp\left(-\left[1.5^{3\delta}\left[\left(\frac{\Gamma_{-1/3}\pi}{\Gamma_{+1/3}}\right)^3\frac{4\Gamma_{-1/3}}{3}\right]^3\frac{\alpha(n)}{2\alpha_{\lim}}\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right]\right)$$

$$\approx \exp\left(-\left[1.5^{3\delta}\left[\left(\frac{\Gamma_{-1/3}\pi}{\Gamma_{+1/3}}\right)^3\frac{4\Gamma_{-1/3}}{3}\right]^3 2\left(\frac{\Gamma_{+1/3}}{\Gamma_{-1/3}2\pi}\right)^3\left(\frac{3}{2}\right)^3\boldsymbol{\Pi_{k=0}^{n}\alpha\wedge(9/3^k)}\left(\frac{e_c}{4\pi\varepsilon_c r}\right)^3\right]\right) \approx \qquad (73)$$

$$\left(\exp\left(-\left[1.5^{3\delta}\frac{\pi^2\Gamma_{-1/3}{}^3}{\Gamma_{+1/3}{}^2}\boldsymbol{\Pi_{k=0}^{n}\alpha\wedge(3/3^k)}\frac{e_c}{4\pi\varepsilon_c r}\right]^3\right)\right)^2 \qquad n = \{0;1;2;..\}$$

47 The need of $\Gamma_{+1/3}/\Gamma_{-1/3}$ to appear in (69)ff and its more pronounced relationship with angular terms is the reason to prefer $(2\alpha_{lim})^{-1} \approx 2(\Gamma_{+1/3}/(2\pi\Gamma_{-1/3}))^3$ over $(2\alpha_{lim})^{-1} \approx 2(2/(2\pi 3))^3$ which would give σ_0=1.821E+8[-], i.e. a term very close to the value of σ_0 fitted to S_z.
48 Expression intended to emphasize 3rd power relationship, a remaining factor of 2 is attributed to $e^{v/2}$ being squared.

Inserted in the equation for energy, (12)f, follows:

$$W_n = 2b_0\int_0^{r_n}\left(\exp\left(-\left[1.5^{3\delta}\frac{\pi^2\Gamma_{-1/3}{}^3}{\Gamma_{+1/3}{}^2}\mathbf{\Pi_{k=0}^{n}}\boldsymbol{\alpha}\wedge\left(\mathbf{3/3^{\mathit{k}}}\right)\frac{e_c}{4\pi\varepsilon_c r}\right]^3\right)\right)^2 r^{-2}\ dr \quad => \qquad n = \{0;1;2;..\} \qquad (74)$$

$$W_\mu = 2e_c\frac{\Gamma_{+1/3}}{3}\ 2^{-1/3}\left[\frac{\Gamma_{+1/3}{}^2}{\pi^2\Gamma_{-1/3}{}^3}\boldsymbol{\alpha}^{\mathbf{-4}}\right] = \frac{2^{2/3}}{3\pi^2}\left(\frac{\Gamma_{+1/3}}{\Gamma_{-1/3}}\right)^3\boldsymbol{\alpha}^{\mathbf{-4}}\ e_c$$

(1.5^δ = extra coefficient for the electron only, $\delta = \delta(0,n)$; bold: particle coefficient; muon given as example)

[A3.3] Relationship with Lorentz boost / (Wigner-) rotation/spin

Interpreting the difference in wavelength of different states as a length contraction due to a Lorentz boost and calculating the necessary velocity according to $l = l_o(1\text{-}v^2/c_0{}^2)^{0.5}$, the ratio of 2 consecutive steps will converge to $v_n/v_{n+1} = 3^{0.5}$ for large n (i.e. small v_n). This is the ratio of the sum of 3 orthogonal vectors of equal length to a single vector, a simple vector addition that corresponds to a Wigner rotation in 3D for the non-relativistic limit [17]. By adding again 3 orthogonal vectors of the resulting vector sum (i.e. of length $3^{0.5}$ of the original vector) one may construct an infinite series of connected states.

The ratio of the vectors, $3^{0.5}$, is the same as that between total spin S and its z-component S_z =1/2, indicating that angular momentum and in particular alignment of magnetic moment / spin of sub-units of particle states may play a role.

The relation according to a Wigner rotation will be less simple for small n. However, the relationship between the length scales $\Gamma_{-1/3}/3\,\rho_e$ and r_l might indicate a similar relation of S_Z and S, involving α and α^3: $\rho_e \sim\sigma^{1/3} \approx 1.5\alpha^{-1}$, $r_e \approx \sigma^{1/3}/2\,\rho_e \sim (1.5\alpha^{-1})^2$, $r_l \approx 2/3\ \alpha^{-1} r_e \sim \alpha^{-3}$. This relation is mirrored in the ratio of coefficients of the electron and muon states.

A relationship such as given in (27) might describe a cascade of interrelated particle states that smoothly transforms into what conventionally would be considered the "field" of a particle.

[A4] Coupling constant in N dimensions

The integration limits for calculating angular momentum in z-direction, r_n of S_z, (15)ff, and (Compton-)wavelength, λ_{Cn}, according to hc_0/W_n should be related by a factor √3, using (13) for W_n and (18), (21)f for r_n plus (36):

$$\lambda_{C,n} = 3\rho hc_0/(2b_0\Gamma_{+1/3}) = 3\pi\,\alpha^{-1}\,\rho/\Gamma_{+1/3};\quad r_n = \alpha^{-1}\,\Gamma_{-1/3}\,\rho/2 \quad => \quad \lambda_{C,n}/r_n = 6\pi/(\Gamma_{+1/3}\Gamma_{-1/3}) = 6\pi/(2\pi\sqrt{3}) = 3^{0.5} \qquad (75)$$

√3 is the coefficient that relates S and S_Z as well. This is not reflected in the length parameter r_l (cf.2.5.3) as equations such as (16), (21)f are highly non-linear as is W(r).

The 3D case of the coupling constant is easy to interpret, for the 4D-case some assumptions have to be made concerning the integration limit. The following gives an alternative, more detailed interpretation than 2.8 ($e^{v(N)} = \exp(-(\rho/r)^N)$).

3D case:

The exact value of the product of the integrals (38)f, depends on the integration limit relevant for the second integral, i.e. the lower integration limit of the Euler integrals, which may be expressed as 3D volume with $\Gamma_{-1/3}$ as radius (18):

$$\rho_n^3/\lambda_{C,n}^3 = 8/\left(3^{1.5}\sigma_0\right) = \left(3^{0.5}\frac{4\pi}{3}\ \Gamma_{-1/3}{}^3\right)^{-3} \qquad (76)$$

The additional factor $3^{0.5}$ may be interpreted as the ratio between r_n of equ. (14) and $\lambda_{C,n}$ as required in the expression for photon *energy*. This gives $\Gamma(-1/3, 1/\sigma_0) \approx 36\pi^2\Gamma_{-1/3}$ and

$$2\int_0^r e^{v(3)}r^{-2}dr\int_0^r e^{v(3)}dr \approx 2\left[\frac{\Gamma_{1/3}}{3}\right]\left[2\pi 2\pi 9\frac{\Gamma_{-1/3}}{3}\right] = 4\pi\,\Gamma_{1/3}\Gamma_{-1/3}\ \mathbf{2\pi} = \mathbf{2\pi}\ \alpha^{-1} \qquad (77)$$ [49]

The result of (77) yields a dimensionless constant $\alpha' = h\ c_0\ 4\pi\ \varepsilon/e^2$ and it is a matter of choice to include 2π in the dimensionless coupling constant. Factor 9 cancels the corresponding factors from the Euler integrals. The remaining factor of 4π is needed to yield the correct value of α.

A general N-dimensional version of (76) may be given as:

$$8/\sigma_N = \left(3^{0.5\delta}V_N\ \left(\Gamma(\text{-}\,1/\mathrm{N})\right)^N\right)^{-N/(N-2)} \qquad (78)$$

V_N is the coefficient for volume in N-D, coefficient $3^{0.5}$ will be omitted in 4D where coordinate r is assumed to be directly related to energy via $r_n \sim 1/W_n$ and r_n might be directly identified with $\lambda_{C,n}$; subscript in σ_N corresponds to dimension in the following.

4D case:

Using $e^{v(4)}$ according to the definition (5) and (78) for 4D:

49 Factor 2 from adding electric and magnetic contributions to energy;

$$\rho_n^4/r_n^4 \;=\; 8/\sigma_4 \;=\; \left(\frac{\pi^2}{2}\;\left(\Gamma_{-1/4}\right)^4\right)^{-2} \;=\; 1.232\text{E-}7 \qquad (79)$$

as integration limit, with (11) the non-point-charge integral in 4D will be given by:

$$\int_0^r e^{v(4)} r\,dr \;\sim\; \Gamma\left(-1/2\,,8/\sigma_4\right) \;=\; \int_{8/\sigma_4}^{\infty} t^{-1.5} e^{-t}\,dt \;=\; 5687 \;\approx\; 16\,\pi^4\,\Gamma_{-1/2} \qquad (80)$$

The 4D equivalent of (77) will be:

$$2\int_0^r e^{v(4)} r^{-3}\,dr \int_0^r e^{v(4)} r\,dr \;\approx\; 2\left[\frac{\Gamma_{1/2}}{4}\right]\left[16\pi^4\,\frac{\Gamma_{-1/2}}{4}\right] \;=\; \frac{\pi^2}{2}\,\Gamma_{1/2}\Gamma_{-1/2}\;\mathbf{4\pi^2} \;=\; \pi^3\,\mathbf{4\pi^2} \;=\; \alpha_{weak}^{-1}\,\mathbf{4\pi^2} \qquad (81)$$

The interpretation is the same as in the 3D-case:
A $4\pi^2$ term originating from the second integral of equation (81) is required for turning h^2 into $\hbar^2$ since the integral refers to ρ_n^2 and thus to the square of energy and h, ħ. Factor 16 cancels the corresponding factors from the Euler integrals. The remaining factor of $\pi^2/2$ is needed to yield the correct value of α_{weak}.

2D case:
the 2D case is not as straightforward as the 4D case. The integral over the 1D point charge

$$\int_0^r e^{v(2)} r^{-1}\,dr \;=\; \Gamma\left(0\,,\rho_n^2/r_2^2\right)\;/2 \qquad (82)$$

features Γ(0, x), with Γ(0, x) -> ∞ for x -> 0 and m = N-2 = 0 in the equations above. Setting nevertheless m=1 in the 2D equivalent of the integration limit

$$\rho_n^2/\lambda_{C,n}^2 \;=\; 8/\left(\sigma_2\right) \;=\; \left(3^{0.5}\pi\;\;\Gamma_{-1/2}{}^2\right)^{-2} \;\approx\; 1/4676 \qquad (83)$$

and calculating $\Gamma(0,\, \rho_2^2/r_2^2)$ numerically gives $\int e^{v(2)} r^{-1}\,dr \approx \Gamma(0,\, \rho_2^2/r_2^2)/2 = 7.872/2$. In the 2D case the complementary integral would be identical to the point charge integral, giving $2(\int e^{v(2)} r^{-1}\,dr)^2 \approx 4\pi^3/4 = \pi^3$, i.e. the same value as 4D, maybe giving an alternate candidate for α_{weak} .

[A5] Quaternion-based quark-like model
[A5.1] Quaternion UDS-components

In the following the model described in chpt. 3 will be explained in some more detail. A standard algorithm for rotation with quaternions will be used.
Three orthonormal vectors E, B, C described as imaginary part of a quaternion with real parts set to 0, will be subject to alternate, incremental rotations around the axes E, B and C. For each E, B and C the following variables will be defined:

- de, db, dc: incremental step for rotation angle,
- de_sum, db_sum, dc_sum: total rotation angle,
- ex, ey, ez, bx, by, bz, cx, cy, cz: Cartesian components of the respective vectors,
- eex, eey, eez, bbx, bby, bbz, ccx, ccy, ccz: Cartesian components of the respective vectors to be buffered until rotation around the axes E, B and C is complete,
- sih, qw, qx, qy, qz: internal variables for quaternion-rotation calculation.

The following part of the algorithm gives the rotation of B around the E axis for an incremental step de:

```
de_sum = de_sum + de;   sih = sin(de / 2);   qw = cos(de / 2);    qx = ex * sih    qy = ey * sih;    qz = ez * sih;
bx = bbx;    by = bby;    bz = bbz;
bxx = bx * (qx * qx + qw * qw - qy * qy - qz * qz)  + by * (2 * qx * qy - 2 * qw * qz) + bz * (2 * qx * qz + 2 * qw * qy);
byy = bx * (2 * qw * qz + 2 * qx * qy)   + by * (qw * qw - qx * qx + qy * qy - qz * qz) + bz * (-2 * qw * qx + 2 * qy * qz);
bzz = bx * (-2 * qw * qy + 2 * qx * qz)  + by * (2 * qw * qx + 2 * qy * qz) + bz * (qw * qw - qx * qx - qy * qy + qz * qz);
bx = bxx;  by = byy;  bz = bzz;
```

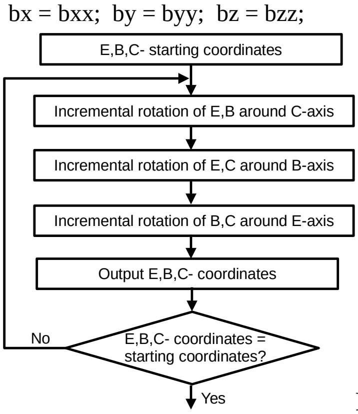


Fig. 4: Flowchart quaternion calculation

This has to be followed by rotation of C around the E axis; and equivalent routines for the rotation of E, B around the C axis and the rotation of E, C around the B axis. After each incremental step for de, db, dc the Cartesian components of the E, B, C vectors may be stored in a list.

The vectors are thought to indicate spatial orientation only, *polarity (sign) of E and B has to be considered in the analysis of the results*. Orientation of angular momentum remains a free parameter.
In the following only solutions where one of the incremental angles of rotation has half the value of the other two will be considered. This may serve as a primitive model for spin S = 1/2.
There are 6 possible solutions for de, db and dv, respectively, to be called U, D, S, C, B, T:

	de = 0.5 db = 0.5 dc				de = 0.5 db = 0.5 dc				de = 0.5 db = 0.5 dc			
	E-comp	E-avg	B-comp	B-avg	E-comp	E-avg	B-comp	B-avg	E-comp	E-avg	B-comp	B-avg
Spherical cone	2/9, 2/9, 1/9	1/3	4/9, 4/9, 2/9	2/3	4/9, 4/9, 2/9	2/3	2/9, 2/9, 1/9	1/3	4/9, 4/9, 2/9	2/3	4/9, 4/9, 2/9	2/3
	U				D				S			
Toroidal wedge	4/9, 4/9, 2/9	2/3	2/9, 2/9, 1/9	1/3	2/9, 2/9, 1/9	1/3	4/9, 4/9, 2/9	2/3	2/9, 2/9, 1/9	1/3	2/9, 2/9, 1/9	1/3
	C				B				T			

Tab.4: Average of x,y,z-components (E,B-comp) and total average (E,B-avg) of E-and B-field for complete rotation;

The average of the x, y, z-components of the fields are multiples of 1/9th of the original vector length, the average total sum of E- and B-fields is 1/3 or 2/3. Surface area / fractional charge of 1/3 and 2/3 correspond to an average of the E-field of 2/3 and 1/3.
The diagram for the E,B, C-components as function of the angle dc_sum is given in fig. 5a for a U-entity.
From a coordinate transformation to a representation with one Cartesian coordinate as axis of rotation (in fig. 5b transformation of z-axis +26,6°, x-axis -41,8°, to give y-axis as axis of rotation) one can infer that the E-vector circumvents a spherical cap of area 2πr (2/3)r. Mirroring at the center of rotation gives a value of 2/3 of the surface of a sphere, which according to Gauss' law may represent 2/3 of a full point charge. The analogue procedure yields a value of 1/3 of a point charge for D and S-rotations.

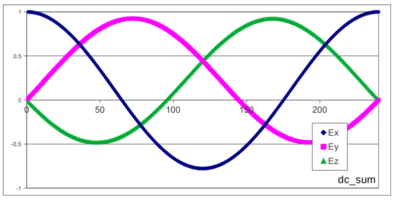
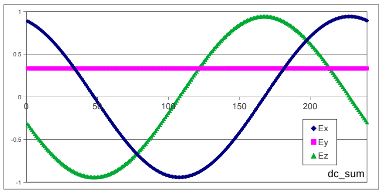


Fig. 5.: a) E-components for Cartesian starting values



b) E-components after coordinate transformation

[A5.2] Magnetic moments of baryons from U, D, S-components [50]

To calculate magnetic moments of uds-baryons three components of U,D,S will be combined that represent orthonormal starting conditions for E, B. Spin/angular moment of the 3 components has to add up to $S_Z = 1/2$. Within this model this is not an assumption but may be calculated in principle in detail. In the following it will be sufficient to have two components sharing the same orientation of the axis of rotation, i.e. both can be transformed according to fig. 5 above with the same set of rotation angles, or - in a trivial case – to have 2 identical components. Together with the freedom in choosing direction of rotation, allowing for cancelling or adding up spin as needed, this will be sufficient to model S_Z = 1/2 baryons. Table 5 gives an example for UUD and DDU.

In D_inv and U_inv the sign of E- and B-components is inverted. The D and U for calculation of the effective B-field include the appropriate sign from their charge while U_inv, D_inv components represent the actual geometric orientation of the E, B-vector only, which is needed for calculation of the angular momentum S from the square of the electromagnetic fields. In table 5 "Rot_X_axis" and "Rot_Z_axis" give the angle of rotation needed to transform to a representation with y-coordinate as axis of rotation for the B-field. For U_1 and D_inv of the proton as well as for D_2 and U_inv of the neutron the angles of transformation are identical, so is their transformed y-axis, i.e. they posses identical orientation of spin (average of B) while still maintaining their orthonormal relationship (B(t)). Since orientation of rotation is a free parameter opposite spin may cancel both contributions, leaving the 3rd component's spin of $S_Z = 1/2$ as total spin of the nucleon.
The U and D components of proton / neutron have equal sign and relative value of the components of the E- and B-fields (given in tab. 5 only for the Bx, By, Bz-components (bold) relevant for calculating a geometry-based average value of B, B_Avg). The results for U and D are exceptional in regard to this exchangeability of U and D-components. For other particle pairs this is difficult to asses due to identical B-field components of U and S and the different internal symmetry of S-components compared to U and D [51].
In the case of the solutions examined, compliance with condition $S_Z = 1/2$ for the lambda-particle (UDS) can be maintained by using a spin-cancelling UD-solution in combination with an S-component, for UUS, DDS, USS-

50 Note: to allow for comparison with tabulated values of M in units of [Am^2] the calculations in this chapter and in chpt. 3.2 use e [C] not e_c [J], conversion factor: [m^2C/s] /[$m^2 J/s$] = e/e_c = 1/19.4 [C/J].
51 U and D are symmetric in their E and B-fields while in S-components E- and B-fields are symmetric to each other.

combinations trivial solutions with two identical components exist, in the case of DSS, Xi$^-$, one might resort to the method used for the nucleons to find a $S_Z = 1/2$ solution. Results for the best fitting appropriate UDS-combinations are shown in tab. 6.

	UUD	**Proton**			**DDU**	**Neutron**	
	U_1				**D_1**		
Start value	-Ez	-Bx	Cy		-Ex	-Bz	Cy
Bx, By, Bz	**-0.444444**	**0.444444**	**-0.222222**		**-0.222222**	**0.222222**	**-0.111111**
	E	B			E	B	
Rot_Z_axis	-45	135			-45	135	
Rot_X_axis	19.47	19.47			19.47	19.47	
	U_2				**D_2**		
Start value	-Ex	By	-Cz		Ey	-Bx	-Cz
Bx, By, Bz	**-0.222222**	**0.444444**	**-0.444444**		**-0.111111**	**0.222222**	**-0.222222**
	E	B			E	B	
Rot_Z_axis	-26.57	116.56			-26.57	116.56	
Rot_X_axis	41.82	41.81			41.82	41.81	
E, B inverted	**D_inv**				**U_inv**		
Start value	-Ey	-Bz	Cx		-Ez	-By	Cx
	E	B			E	B	
Rot_Z_axis	-45	135			-26.57	116.56	
Rot_X_axis	19.47	19.47			41.82	41.82	
	D				**U**		
Start value	Ey	Bz	Cx		Ez	By	Cx
Bx, By, Bz	**0.222222**	**0.222222**	**0.111111**		**0.444444**	**0.444444**	**0.222222**
Bx, By, Bz Avg(UUD)	-0.148148	0.37037	-0.185185		0.037037	0.296296	-0.037037
B_Avg			**0.439790**				**0.300890**

Table 5: Example for appropriate combinations of U- and D-components for proton and neutron;

	USD	**Lambda**		**UUS**	**Sigma +**		**DDS**	**Sigma -**		**USS**	**Xi 0**		**DSS**	**Xi -**	
	U			**U**			**D**			**S**			**S**		
Bx, By, Bz	-0.444	0.444	-0.222	-0.222	0.4444	-0.444	-0.111	-0.222	0.222	-0.222	-0.444	-0.444	0.444	-0.222	0.444
	S			**U**			**D**			**S**			**S**		
Bx, By, Bz	0.444	-0.444	0.222	-0.222	0.4444	-0.444	-0.111	-0.222	0.222	-0.222	-0.444	-0.444	-0.444	0.444	-0.222
	D			**S**			**S**			**U**			**D**		
Bx, By, Bz	0.222	0.222	0.111	0.4444	0.4444	0.222	0.444	0.444	0.222	0.444	0.444	0.222	0.222	-0.222	0.111
Bx, By, Bz Avg(UUD)	0.074	0.074	0.037	0.000	0.444	-0.222	0.074	0.000	0.222	0.000	-0.148	-0.222	0.074	0.000	0.111
B_Avg			**0.111**			**0.497**			**0.234**			**0.267**			**0.134**

Table 6: Combinations of UDS-components for calculating magnetic moments of baryons.

To calculate magnetic moments, above factors of B_avg, derived from the purely geometric quaternion model, have to be multiplied by a factor considering the absolute strength of fields. Using a simple model for a current loop, M = I*S (current * area), gives equ. (84) for magnetic moments of baryons with $S_Z = 1/2$.

$$\boldsymbol{M_n} \approx e c_0 \lambda_C / 2 \; * \; \text{B\_avg} \quad (= 2\pi \mu_B * \text{B\_avg}) \tag{84}$$

see tab. 7. Factor 2π in the Bohr magneton, μ_B, applicable for the electron and muon, is considered to represent a degree of rotational freedom of simple particles that more complex structures composed of several U, D, S-components do not exhibit and thus 2π has to be cancelled.

Results of table 7 are obtained from a large set of solutions, thus the statistical significance is low and a more comprehensive study of the appropriate combination of spin-components is needed. Control samples have been made to

		λ_C	e c_0 *λ_C /2	B_Avg	|M|Calc = $ec_0\lambda_C$ B_{avg}/2	|M|Exp[Am^2]	|M|Calc/ |M|Exp	|M|Calc/|M|Exp Const. quark
p^{+-}	UUD	1.32E-15	3.17E-26	0.440	1.39E-26	1.41E-26	0.988	-
n	DDU	1.32E-15	3.17E-26	0.301	9.55E-27	9.66E-27	0.988	0.973*
Λ^0	UDS	1.10E-15	2.64E-26	0.111	2.94E-27	3.10E-27	0.949	-
Σ^+	UUS	1.04E-15	2.50E-26	0.497	1.24E-26	1.24E-26	1.002	1.090
Σ^-	DDS	1.04E-15	2.50E-26	0.234	5.83E-27	5.86E-27	0.994	0.897
Ξ^0	USS	9.43E-16	2.26E-26	0.267	6.05E-27	6.31E-27	0.958	1.152
Ξ^-	DSS	9.38E-16	2.25E-26	0.134	3.01E-27	3.06E-27	0.983	0.784

Table 7: Magnetic moments for UDS-Baryons; col.3: Compton wavelength [7]; col.4: magnetic moment for current loop; col.5: average B-component from quaternion calc.; col.6: calculated magnetic moments; col.7: values from experiment [7]; col.8: ratio calculated / experiment value; col.9: ratio (calculated constituent quark model, [7]) / experiment [7]), *calc. via Clebsch-Gordan coefficients relative to p; Σ, Ξ via fit based on p, n, Λ^0.

check that a) in rare cases where U and D solutions do not match a UUD/DDU pair the condition for S = 1/2 is not met; b) all U and D- components shown in the tables in combination with an S are components appearing in UUD/DDU-pairs as well.

[A5.3] Ratio of magnetic moments

The calculation of the ratio of magnetic moments is particularly simple and may be based on geometry only. In the quaternion model both E- and B-fields are oriented to the center (magnetic monopole character on particle level) and will feature average fields of 1/3 and 2/3 for quark-like objects. The B-field for u- and d-entities will have Cartesian components of ± 2/9, ± 2/9, ± 1/9 (d) and ± 4/9, ± 4/9, ± 2/9 (u). Unique solutions [52] for B-field components of nucleons will be e.g. ($B_{avg}=((\sum x_i)^2+(\sum y_i)^2+(\sum z_i)^2)^{0.5}/3$):

proton - uud -4/9, -4/9, -2/9 / -2/9, -4/9, -4/9 / +2/9, -2/9, +1/9 $B_{avg} = 141^{0.5}/27 \approx 0.440$

neutron - ddu -2/9, -2/9, -1/9 / -1/9, -2/9, -2/9 / +4/9, -4/9, +2/9 $B_{avg} = 66^{0.5}/27 \approx 0.301$

The ratio of both values is $(141/66)^{0.5}$ = 1.461631, which compared to the ratio from experiments [7] gives 1.461631 / 1.459898 = 1.001187.

These solutions are distinguished by one U and one D-component being collinear [53], indicating a particular stable configuration involving oppositely charged components (see [A6]).

Table 8 compares some ratios of baryon isospin pairs for calculations with the average of the B-field as calculated in [A5.2] with B_avg, i.e. geometry only, and with the experimental value of the Compton wavelength/particle energy.

	U,D,S-components	$\lvert M\rvert$Calc (λ_C exp)	B_avg
M(p/n)_Calc/M(p/n)_Exp	UUD/DDU	0.999809	1.001187
M(Σ^+/Σ^-)_Calc/M(Σ^+/Σ^-)_Exp	UUS/DDS	1.007813	1.001111
M(Ξ^0/Ξ^-)_Calc/M(Ξ^0/Ξ^-)_Exp	USS/DSS	0.974652	0.969601

Table 8: Ratio of particle magnetic moments of baryon isospin pairs compared for calculated and experimental values [7] (col.3: geometry only, B_avg; col.2 inc. experimental particle energy);

[A6] Nucleons – stability, bonding in nuclei, scattering

Apart from the quantitative results for partial charges and magnetic moments some qualitative trends for nucleon properties may be inferred from the quaternion-based model.

The spin-cancelling of a UD-unit involves 2 collinear components with opposite charges occupying approximately the same spatial area (fig. 6), which is energetically favorable. This suggests among other things:

1) Comparatively lower energy for particles with UD-component;
2) High stability / life time of the nucleons;
3) A possible contribution to bonding in nuclei via UD-U—D-UD, a direct U-D-bond even without meson intermediate;
4) If such an inter-nucleon UD-bond plays a role in bonding in nuclei this would suggest a significant change in UD-structure between isolated and bound nucleons, which might play a role in the "EMC-effect" [18];
5) In DIS-experiments the ratio of the structure functions of neutron and proton, $F_2^n(x)/F_2^p(x)$ approaches 1 for x -> 0 (x = Bjorken-scale) which would be in agreement with the resolution of identical E and B fields of the EBC-triple of the nucleons rather than the averages of their U or D-units. For x -> 1 this model predicts the ratio $F_2^n(x)/F_2^p(x)$ to approach
$(z(UD)^2 + Z(D)^2)/(z(UD)^2 + Z(U)^2) = ((+1/3)^2 + (-1/3)^2)/((+1/3)^2 + (+2/3)^2) = 2/5$
which is in good agreement with high precision scattering experiments that yield values in the range 0.4 – 0.5 [19].

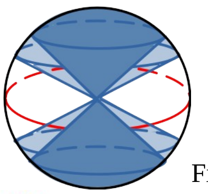


Fig. 6: Illustration of a UD-unit

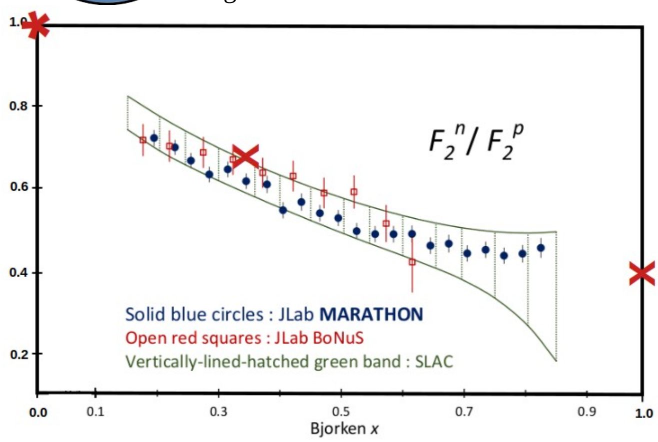


Fig. 7: Ratio of nucleon structure functions (adapted from [19]); red crosses: values according to partial charges only, at x≈1/3 according to U+U+D and D+D+U units (2/3), at x=1 according to UD-units (2/5); Star at x=0 corresponds to an identical field distribution of E and B-fields in the nucleons, where the time average of E and B fields resulting in structures such as given in fig. 6 will be replaced by E(t) and B(t), both of identical strength for U and D.

52 same permutations and signs for u- and d-components; unique except for arbitrary orientation in space;
53 Time average! All E,B-components involved are orthogonal at any given point in time.

[A7] Additional particle states

Assignment of more particle states will not be obvious. The following gives some possible approaches.

[A7.1] Partial products

One more partial product might be inferred from considering the next spherical harmonic, y_2^0 with a factor of $(2l+1)^{1/3}$ = $5^{1/3}$ as energy ratio relative to η, giving the start of an additional partial product series at $5^{1/3}$ W(η) = 937MeV i.e. close to energy values of the first particles available as starting point, η', Φ^0. However, in general it is not expected that partial products can explain all values of particle energies.

[A7.2] Linear combinations

Though the model reproduces basic properties of the quarks the fundamental differences might offer some alternate interpretations based on extended, non-point-like objects.

Linear combination states of the kaons, the first particle family that does not fit to the partial product series scheme, (31), and the η-particle might be an example for such an interpretation:

The kaons are designated to the linear combination of $(d\bar{s} +/- \bar{d}s)/\sqrt{2}$ in the SM. They might be considered to be a linear combination of 2 extended y_1^0 states (double cones of $s|\bar{d}$, $\bar{s}|d$, etc., composition with 1 angular node) similar to the linear combination of 2 atomic p-orbitals, assumed to exhibit 2 angular nodes. A linear combination which would yield the basic symmetry properties of the 2 neutral kaons would be a planar structure such as:

		$\bar{s}$				d	
K_S^o	d		s	K_L^o	s		$\bar{s}$
		$\bar{d}$				$\bar{d}$	

providing two neutral kaons of different structure and parity (considering either flavour or chirality), implying a decay with different parity and lifetime [54].

A linear combination of 3 such states i.e. 3 orthogonal y_1^0 states would imply an approximate spherical symmetric object which might be attributable to the η-particle $((u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6})$.

[A7.3] Higgs boson

The considerations of chpt. 2.6 give the Higgs VEV as upper limit, the Higgs boson has about half its energy value. The "rotating E-vector" of chpt. 3 may be interpreted to cover the whole angular range in the case of y_0^0 of e.g. e or μ, while a y_1^0 object might be interpreted as forming a double cone. Increasing the number of angular nodes would close the angle of the cone leaving in the limit l -> ∞, a state of minimal angular extension representing the original E-vector. This may imply that essentially no space is left for rotation (i.e. Spin = 0) and a vanishing contribution of the magnetic field to total particle energy according to (13), resulting in a factor 1/2 and giving the Higgs *boson* as alternate upper limit of energy.

54 In analogy, for the charged Kaons, K^+, K^-, two versions of P+ and P- parity of otherwise identical particles might be possible if one exchanges e.g. $(d,\bar{d})$ => (u,u) or $(\bar{u},\bar{u})$ and $(s,\bar{s})$ => $(\bar{s},\bar{s})$ or (s,s) in the configurations above.